STRUCTURE INDEPENDENT SEISMIC VELOCITY ESTIMATION

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DOCTOR OF PHILOSOPHY

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Many commonly used seismic velocity estimation procedures assume that the reflectors are horizontal. Because of this their performance tends to degrade as the reflectors become curved or discontinuous. Much of this degradation can be traced to the fact that data recorded over non-horizontal reflectors need not resemble in detail the subsurface in the area where they were recorded. Diffraction and scattering are the major complicating factors.

Using general principles of reflector mapping it can be shown that surface recorded seismic reflections which have been downward continued to the depth of their source reflectors must resemble those reflectors in detail. This property of downward continuation can be exploited to improve velocity estimates by using downward continuation as a pre-processor for velocity estimation techniques. The estimates resulting from this kind of approach should not exhibit diffraction effects and should not be dependent on reflector dip.

Since much reflection seismic data are described to a good approximation by the scalar wave equation, this equation is an obvious starting place in deriving a downward continuation operator for seismic data. Beginning with the scalar wave equation and using a small dip assumption, approximate wave equations which quite accurately model both near and wide angle reflections generated by one or more sources can be found. Finite difference formulations of these equations can be used as stable and economic downward continuation operators for reflection data.
These wave equation continuation operators allow the demonstration, with both synthetic and field data examples, that downward continuation represents an economically viable process for removing the effects of reflector geometry from seismic velocity estimates. Additionally, synthetic data examples illustrate the fact that the use of downward continuation allows accurate velocity estimates to be made from no-record data recorded over an earth in which the reflectors are random functions of the horizontal and vertical coordinates. For reasonable data parameters, theoretical considerations indicate that the semblance of properly downward continued random reflector data measured along the true velocity hyperbolic is approximately 50% greater than a similar measure on the corresponding surface data. This semblance increase should make velocity estimates based on downward continued random reflector data less susceptible to noise than estimates based on surface recorded random reflector data.

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Chapter 1. Introduction

The accurate estimation of velocity from reflection seismic data has long been a subject of interest to explorationists. Most commonly used velocity estimators are based on a layered media assumption. Because of this, their performance tends to degrade as the earth becomes non-layered. Here a method which allows accurate velocity estimates to be made from data recorded over a particular type of non-layered earth is presented.

Figure 1-1 indicates the two main ways in which the earth deviates from the usually assumed plane layers. One type of deviation occurs when velocity is constant or layered but the reflectors have arbitrary dip and curvature. Another occurs when the reflectors are planar but the velocity structure is arbitrary. The second class of structures tends to have the more drastic effect on velocity estimators. Although structures of the first class cause less drastic degradation of velocity estimates, there are situations where accommodation of their effects are of great importance. In this thesis we shall restrict our study of velocity estimation to data recorded over first class models.

Many of the difficulties associated with the estimation of velocity from seismic data recorded over non-horizontal reflectors can be traced to the fact that such data need not resemble, in detail, the earth structure over which they were recorded. (Diffraction and scattering are the major complicating factors.) Migration can be defined as an operation which suppresses diffraction and scattering effects by reorganizing the reflection data so that they resemble a reflectivity map of subsurface over which they were recorded. Because migration can be used to remove the phenomena which cause most of the velocity estimation difficulties in the earth models we
Figure 1-1. Earth structures for which velocity estimation may be difficult.

The left frame shows an earth in which the velocity is layered but the reflectors have arbitrary dip and curvature. The right frame shows an earth in which the reflectors are horizontal planes but the velocity structure is arbitrary. Realistic earth models should contain both types of structures. The methods we describe here are strictly valid for structures like those on the left. However, they should have some applicability to structures like those on the right.
are considering, velocity estimates made from migrated data should be superior to those made from unmigrated data. The main objective of this thesis is to show that this improvement does occur and that the practical problems associated with migrating data prior to the determination of true velocities can be overcome.

In this study we shall use wave equation methods to perform our migrations. We have chosen to use wave equation techniques (Claerbout and Doherty, 1972) rather than the ray techniques (Peterson and Walter, 1974) because there are some indications that practical implementations of the wave methods may produce results which are superior to those obtainable with ray techniques. However, our choice of a particular approach toward migration does not mean that velocity estimates can be improved only if that approach is used. As long as care is used in performing migrations, velocity estimates should have only a marginal dependence on the migration approach used.

In the following chapter some terminology and techniques often used in reflection seismic exploration are presented. There is also a brief description of some prevalent methods of velocity estimation. This chapter is intended as tutorial information for those not familiar with seismic exploration. Old hands, and doodlebuggers may wish to skip on to chapter 3.

In chapter 3 we begin a discussion of wave equation migration as it applies to wave fields generated by a single source. Although much of this material has been published previously it has been included here as an introduction to the problem of migrating data generated by many sources. Many of the assumptions and techniques developed in chapter 3 will be used extensively in the following chapters. A synthetic example has been included to illustrate wave equation migration and to show how the equations
presented in chapter 3 extend the results of the previously published work.

In chapter 4 we tackle the many source problem which is essential to reflection seismic velocity estimation. We investigate the new characteristics of the migration equation associated with multiple sources. We find that if source-receiver directivity effects are ignored and small dip and moderate source-receiver offset assumptions are adopted, multiple source data can be migrated with equations no more complicated than those required for the single source problem.

In chapter 5 the equations and concepts developed in earlier chapters are applied to the velocity estimation problem. First we discuss, in some detail, the effects of reflector structure on velocity estimates. We then demonstrate that migration with the known velocity can accommodate these effects and allow accurate velocity estimation. Additionally, we show that satisfactory results can be obtained in the case where the correct migration velocity is not known. Finally, we apply the method to the problem of estimating velocity in regions where the earth has little or no structural continuity.
Chapter 2. Basic Concepts from Reflection Seismology

Introduction

In this chapter we shall present some of the basic concepts and techniques that are used in reflection seismic exploration. We shall also introduce terminology that will be used in later chapters. We shall mainly be concerned with field recording geometries, data display coordinates and simple velocity estimation techniques.

Velocity Estimation

Consider the experiment shown in Figure 2-1. A single source, s, generates acoustic waves which reflect at the interface and are recorded at the receivers, g (geophone). The recorded seismograms are plotted at the right as a function of shot-receiver offset, (g-s). We define this recording geometry as the profile geometry. The data display will be termed a profile. The arrival times of profile reflections follow the well known hyperbolic trajectories given by

\[ t^2(\text{g-s}) = t^2(0) + \frac{(\text{g-s})^2}{v^2} \]  

(2-1)

where \( t(0) \) is the two way vertical travel time and \( v \) is the velocity of the medium above the reflector. Since there are many arrival times and only two unknowns ( \( t(0) \) and \( v \) ), these profile data can be used to determine the seismic velocity in the medium above the reflector. The profile is the simplest type of experiment which can be made to determine velocity.

Unfortunately, because profile arrival times are strongly dependent on reflector dip and curvature, velocity estimates from profiles are unreliable. The effects of reflector dip can be reduced somewhat, if estimates are made from data recorded in the geometry shown in the left frame of Figure 2-2. Notice that unlike the profile, the geometry in
Figure 2-1. The profile recording geometry. The arrival times of the reflections are shown at the right. The data display is called a profile.

Figure 2-2. Common midpoint gather geometry and data.
Figure 2-2 has many shots and many receivers. Notice also that each shot-receiver pair has the same midpoint, \( \frac{(g+s)}{2} \). The seismograms recorded in this geometry are plotted at the right as a function of shot receiver offset. We shall term this type of display a common midpoint (CMP) gather. Often this data display is called a common depth point gather. We shall not use that terminology here, because such data have a common depth (reflection) point only if the reflectors are plane layered. The arrival times of the CMP gather data are also given by equation (2-1). Unlike profiles, data recorded in this geometry are hyperbolic even when the reflector is only locally horizontal at the reflection point. The common midpoint gather is the basic data used by most of the prevalent velocity estimators [Schneider and Backus (1968), Tanner and Koehler (1969), Sherwood and Poe (1972)].

Velocity estimation from CMP gathers amounts to determining which values of \( t(0) \) and \( v \) best fit the arrivals displayed on the gather. Typically the determination of the best fitting parameters is made by actually trying many possible values of \( v \) for each \( t(0) \) and then picking the velocity which fits best. The main differences between velocity estimators lie in the method of determining how well each trial velocity fits the data. Some methods use unnormalized cross correlation, others use semblance and still others use summing techniques. Figure 3 shows how velocity might be estimated using a summation measure. The estimated velocity depends to some extent on the coherence measure used, however for our purposes we can consider estimates to be independent of the coherence measure.

Up to this point we have discussed velocity estimation in terms of reflections from a single layer. Dix (1955) showed that for small
Figure 2-3. An example of velocity estimation. On the left is a CMP gather. The arrivals shown represent a plane layer reflection event. The dotted and dashed hyperbolas represent three sets of arrival times corresponding to three different velocities but having the same zero offset travel time. For the data shown here, velocity estimation amounts to determining which hyperbola best fits the event on the gather. Here, the power in a sum of the data along each hyperbola is used as a measure of goodness of fit. The graph on the right shows the power resulting from a sum of the data along each of the trajectories shown (others too). Since peak power occurs for lag pattern 2, the corresponding velocity, \( v_2 \), would be the velocity estimate of this procedure. In this example we have summed only the data corresponding to zero offset travel time \( t_0 \). In practice, the power in the hyperbola sums for all data having zero offset travel times within a gate around \( t_0 \) would be averaged in getting a velocity estimate for time \( t_0 \). Averaging is used to suppress noise and to overcome the effects of finite length source wavelets.
shot-receiver offset, the reflections recorded over multi-layered velocity structures have arrival times which approximately satisfy equation (2-1). He also showed that for those reflections the best fit velocity in equation (2-1) was a type of average of the velocities of the layers above the interface causing the reflection. Specifically, the best fit velocity for the reflection from the bottom of the $N^{th}$ layer is given by

$$v = \left[ \frac{\sum_{i=1}^{N} v_i^2 \Delta t_i}{\sum_{i=1}^{N} \Delta t_i} \right]^{1/2}$$

(2-2)

where $v_i$ is the velocity and $\Delta t_i$ is the two way vertical travel time in the $i^{th}$ layer above the reflector.

For obvious reasons the velocity defined by (2-2) is called the rms velocity of the reflection. Shah and Levin (1973) demonstrate that for reasonable recording geometries and velocity structures, velocity error associated with fitting equation (2-1) to reflections from multi-layered media is usually less than 2%.

From the above discussion it should be clear that the basic assumption of the standard velocity estimation procedures is that the arrival times of reflections are hyperbolic when displayed on a CMP gather. We have already seen that this is not exactly true for multi-layered structures. In general, it is also not true when either velocity or reflector geometry is laterally variable.

**Recording Geometries and Data Displays**

As a final topic in this chapter we will discuss recording geometries and data displays. Typically, good quality marine reflection seismic data is recorded with an array consisting of one source and up to 48 or 96 receivers.
This array is towed by the recording vessel along some straight line which we shall call the 'k' axis. The source is activated periodically and the generated reflections are recorded. Data recorded for each source activation are by our previous definition a profile. Thus, as the boat moves along the 'k' axis, a series of profiles each with a different source position and reflection positions, are recorded. The source is activated often enough so that adjacent profiles share many of the same reflection points. Figure 2-4 indicates this type of recording geometry and shows some of the ways these data are normally displayed. The figure shows the spatial coordinates of the source and receivers for the data displayed in profiles, common offset sections and common midpoint gathers. A three dimensional display which contains all the seismograms recorded while the vessel moved along a particular portion of the 'k' axis is shown in Figure 2-5. We shall term this type of display a multi-offset section. Since velocity estimates are usually based on data generated by many sources, our ultimate goal in later chapters will be to develop techniques which are applicable to common offset sections, multi-offset sections and CMP gathers.
Figure 2-4. Recording geometries and data display coordinates. The top drawing shows a typical marine recording geometry. The boat moves to the right along the 'k' axis. Behind, it tows a source array and a cable of receivers. Here we have denoted the source as s and the receivers as g (geophone). The lower diagram shows some typical recording geometries and data displays in (s,g) space (source coordinate,
geophone coordinate). Note that all axes (g,s,x,y,h) are parallel to the boat path (k axis). As the boat translates along the k axis data are collected in the speckled region of the s-g plane. At each source activation reflections are recorded at receivers positioned along the receiver cable. We have darkened the position of the cable for several shot locations. Data recorded and displayed along these lines are called profiles. Data recorded and displayed along lines parallel to the sequence of positions occupied by a given receiver (along lines parallel to the y axis) are called common offset sections. Because each profile shares some common receiver positions data can also be displayed along lines perpendicular to the y axis. Such displays along the offset (h) axis are called common midpoint gathers. There is a fundamental difference between the amount of data that can exist along the midpoint (y) axis and the amount that can exist along the (h) or (x) axis. The maximum (h) or (x) extent of the data is determined completely by the receiver cable length. The maximum length of data in the (y) direction is unrelated to receiver cable length.
Figure 2-5. Data of a multi-offset section. A multi-offset section contains all the seismograms recorded along a particular traverse line. The seismograms of a multi-offset section are parameterized in terms of offset, midpoint and time. Thus, a multi-offset section may be thought of as either a section of common midpoint gathers or as a suite of common offset sections.
Chapter 3. Downward Continuation of Profiles

Introduction

Although profile data are not central to the velocity estimation problems we wish to deal with in this thesis, we will spend some effort to describe the profile problem in the hope that familiarity with profiles will make the somewhat more difficult problem of downward continuation of sections easier. Since profiles are generated with only one source, much observational experience and insight can be readily applied to them. This is not the case for sections. The fact that the wave field to be downward continued is generated by many separate sources often makes it difficult to apply insights gained from other wave phenomena to sections. Another reason for studying profiles as an introduction to downward continuation, is that profiles can be described with one less coordinate than sections.

Downward Continuation and Reflector Mapping

We shall begin with a discussion of what we mean by downward continuation and of why it is important. By downward continuation of seismic data we mean synthesizing data that would be recorded with buried receivers from the data recorded at the earth's surface. Figure 3-1 gives an indication of the desirability of downward continued data. The top frames show the hyperbolic nature of the data recorded with surface receivers over a point scatterer. The bottom frames show the reflections that would be recorded if the receivers were located at the depth of the scatterers instead of at the surface. (Note that we have excluded horizontally propagating waves.) A look at the buried receiver data shows that reflections are recorded only at the receiver positioned on the scatterer. Figure 3-1 illustrates the general statement that data
recorded with buried receivers give a simpler picture of the subsurface than do data recorded at the surface.

Figure 3-la. Profile recorded over a point scatterer. The frame on the left shows the reflection paths. The right frame shows the data. Wave velocity, $v$, is constant.

Figure 3-1b. Profile reflections from a point scatterer recorded with buried receivers. Since we exclude horizontally propagating waves reflections are received only at the geophone located at the point scatterer.
To describe the uses of downward continued data a bit more precisely we shall need to consider reflector mapping in general. The basic principle of reflector mapping is that reflectors exist at points in the earth where the first arrival of the downgoing wave is time coincident with an upcoming wave. In the absence of multiple reflections, this is the only principle needed to map subsurface reflectivity in a region of known velocity. Since data recorded with buried receivers are just the upcoming wave field at the receiver location, downward continued data contain almost all the information necessary to determine reflector geometry. The other information needed is the subsurface downgoing wave. Fortunately, in the absence of multiple reflections, synthesis of the downgoing wave is simple, since to first order it is independent of subsurface reflectivity. One does not err much in assuming that the downgoing wave of profiles can be modelled by a quasi-spherical wave expanding in the velocity structure of interest. Thus, one reason downward continuation is interesting and important to the geophysicist, is that in many cases, the ability to perform downward continuation is equivalent to the ability to map subsurface reflectivity.

The process of transforming seismic data into a map of subsurface reflectivity is usually called migration. The reflectivity maps are often called the migrated data. Although downward continuation has its most obvious application to the field of migration, in later chapters we show that it can be an important tool in velocity estimation.

A final topic we need to discuss in this section is the operator which we should use for downward continuation. The wave equation is the operator which governs propagation of the upcoming wave from the reflector
to the surface receivers. Accordingly, we shall use this same operator, albeit time reversed, to propagate (downward continue) the upcoming wave from the surface back to the reflectors.

**Moveout Correction**

The process of reflector mapping we have described requires downward continuation of the upcoming wave and a determination of where the downgoing and upcoming wave are time coincident. Consider the case where the reflectors are plane layered and velocity is constant. If the downgoing wave is a vertically incident plane wave, the search for time coincidence is simple because neither the downgoing or upcoming wave depends on the horizontal coordinate. This simplicity is lost in the profile geometry because the downgoing wave is spherical. Its arrival time and the arrival time of the reflected wave at a particular receiver depend on both the vertical and the horizontal coordinate of that receiver.

Even if we ignore the question of time coincidence, figure 3-2 shows that downward continuation of profile data will probably be more difficult and expensive than continuation of the reflections generated by a plane wave source. For the constant velocity, flat reflector case, downward continuation of plane wave reflections amounts to simple laterally invariant time shifting. Profile wave field continuation requires hyperbolic time shifting and lateral repositioning of the data.

It would be a great advantage to be able to treat profile wave forms recorded over layered reflectors with the same ease as the wave fields generated by plane wave sources. One way of accomplishing this goal is to perform downward continuation and reflector mapping, in a coordinate system where the profile waveforms recorded over layered reflectors appear planar. To transform the data into these coordinates we shall need
Figure 3-2. Profile and plane wave source data before and after continuation. The bottom frame shows an un-moveout corrected profile recorded over a horizontal reflector. The curvature of the downward continued data (right) is the result of the hyperbolic arrival times of the downgoing wave. The smaller horizontal extent of the downward continued data is due to the fact that reflection points of the surface data are at the midpoint of the surface shot and receiver. The top frame shows the data expected for a plane wave source. The downward continued data differ from the surface data only by a laterally invariant time shift. If moveout correction had been applied to the data of the lower frame they would have appeared exactly like the data of the top frame. Moveout correction requires that surface data be time shifted hyperbolically and that data recorded at each receiver be placed at the midpoint of the shot and that receiver.
equations which deform the hyperbolic arrival times of reflected spherical waves into horizontal lines. Another way of saying this, is that we need equations which correct for the differential travel time caused by non-zero shot receiver offset. Such transformations have long been used by explorationists. They are called normal moveout or just moveout corrections. We adopt that terminology here. Data displayed in these hyperbolically deformed coordinates will be called moveout corrected data.

If the simplicity gained by using moveout corrected data was achieved only when the reflectors were horizontal, moveout correction would be of little value to us. However, moveout correction approximately corrects for the effects of shot receiver offset even when the reflectors are only approximately horizontal. Because of this, it is nearly always true that less work is required to downward continue moveout corrected data than raw data. This reduction is important to us because we shall use numerical methods to perform downward continuation. In general, the less work a numerical scheme must do, the more accurate and inexpensive it becomes.

In addition to reducing numerical-computational problems, the use of moveout corrected data also eases the determination of time coincidence of the upcoming and downgoing wave. To see this, consider a moveout corrected profile recorded over a curved or dipping reflector. Suppose each portion of these data was downward continued until the arrival time of the downward continued data corresponded to the vertical travel time associated with the receiver depth. In this case, the arrival time of the moveout corrected downgoing wave (the vertical travel time) would be the same as the travel time of the upcoming wave (the data). Thus, by our mapping principle the downward continued data would be a
reflectivity map. By using moveout corrected data we replace the task of finding time coincidence of the upcoming and downgoing wave with the simpler task of insuring that the data are continued until their arrival time equals the vertical travel time associated with the receiver depth.

Because of the advantages associated with the use of moveout correction, any subsequent discussion of downward continuation in this thesis will always be in terms of moveout corrected data.

Coordinate System and Wave Equation

Once we decide to downward continue moveout corrected data, we are faced with the question of how the wave equation must be modified so that it is valid for wave fields modified by a moveout correction. To resolve this question we must consider in some detail what we mean by the wave equation and moveout correction.

We will consider the wave equation first. To a great extent the form and complexity of the wave equation depends on the materials in which one wishes to study wave propagation. More correctly, the form one uses does not depend on the materials, but instead on the assumptions about the materials (be they correct or not) one wishes to make. In this thesis we shall assume that the materials are such that wave propagation is adequately described by the scalar wave equation. We shall also assume that no shear wave can exist in the medium. In deriving continuation equations we shall also assume that any reflectors are independent of one horizontal coordinate and thus, we shall use a 2-dimensional wave equation.

The form of the wave equation also depends on the coordinate system in which it is expressed. Generally, we write the wave equation in a system directly relatable to physical space. In this discussion, we shall express it in a 2-dimensional cartesian coordinate system with \( g \) being the horizontal coordinate, \( z \) being the vertical coordinate with \(+z\) down into the earth, and \( t \) being the time coordinate. To
fix the system to the problems of interest we shall assume the receivers are distributed along the g axis and that the shot is located at 
\((g,z,t) = (s,0,0)\). With all these things in mind, we can write the wave equation which we assume governs our problem as

\[
P_{gg} + P_{zz} = \frac{1}{v^2} P_{tt} + \delta(s-g,z,t)
\]

(3-1)

where \(P\) is a pressure and \(\tilde{v}\) is the compressional wave velocity. We have used subscripts to denote partial derivatives. The delta function represents the source.

Now that we have the wave equation tied down, we will discuss moveout correction. As we have said previously, moveout correction is an operation designed to remove hyperbolic, source-receiver geometric effects from the data. An important property of this operation is that it is one-to-one. That is, each point on the field data (the upcoming wave recorded at the earth's surface) is mapped uniquely onto the moveout corrected profile. Because of this one to one property, moveout correction can be thought of as an invertible coordinate transformation from a set of field recording coordinates to a set of moveout corrected coordinates.

We define these moveout corrected coordinates as follows: \(x\) is half the surface source receiver separation, \(d\) is a moveout corrected two-way travel time and \(r\) is the receiver depth. Using these definitions, and referring to Figure 3-3, we can express the normal moveout operation as

\[
x = g/2 \left( 1 + \frac{z}{(v(t^2 - g^2/v^2)^{1/2})} \right)
\]

(3-2a)

\[
d = (t^2 - g^2/v^2)^{1/2} + z/v
\]

(3-2b)

\[
r = z
\]

(3-2c)

where \(v\) is a constant moveout correction velocity which need not be related to \(\tilde{v}\), the velocity in the wave equation.
These equations are an extension of the usual definition of normal moveout because they are depth dependent. For the moment, we will study the effects of moveout correction on surface data by setting $z = 0$. In this case we see that (3-2a) performs the operation of placing each trace at the midpoint of the shot and receiver. Equation (3-2b) performs the hyperbolic time shifts necessary to flatten reflections from plane layers.

The reason that we have explicitly included depth dependence is that we wish to describe moveout corrected data that are recorded at subsurface locations. Since $x$ is usually thought of as the horizontal coordinate of the reflection point, and $d$ as a measure of reflector depth, equations (3-2a) and (3-2b) include a $z$ dependence such that...
for plane layer reflections, these quantities are constant along a ray path. This definition for \( d \) makes this type of moveout correction different than that discussed in the previous section, in that downward continuation in this coordinate system will cause no change in the arrival times of plane layer reflections which have been properly moveout corrected.

As we noted previously, transformation (3-2) is invertible; the

\[
\begin{align*}
g &= (d - r/v) \frac{2x}{d} \\
t &= (d - r/v) \left( 1 + \frac{4x^2}{v^2} \frac{d^2}{2} \right)^{1/2} \\
z &= r
\end{align*}
\]  

(3-3a) (3-3b) (3-3c)

Continuation Equation

Now that we have equations (3-1), (3-2), and (3-3), we return to the question of modification of the wave equation. Since moveout correction is a coordinate transformation expressed by (3-2) and (3-3), all we need to do to find the equation which governs moveout corrected data is to transform (3-1) into the moveout corrected coordinate frame.

As a first step we note that the wave fields are invariant under coordinate transformations. That's

\[ Q(x,r,d) = P(g,z,t) \]  

(3-4)
We have defined $Q$ to be the wave field viewed in the moveout frame.

Now we can transform the wave equation. Using the chain rule we have

$$P_t = Q_x x_t + Q_r r_t + Q_d d_t$$
$$P_{tt} = Q_{xx} x_t^2 + Q_{rr} r_t^2 + Q_{dd} d_t^2 + 2x_t d_t Q_{xd} + 2x_t r_t Q_{xr}$$
$$+ 2 r_t d_t Q_{rd} + x_{tt} Q_x + r_{tt} Q_r + d_{tt} Q_d$$
$$P_{zz} = Q_{xx} z_z^2 + Q_{rr} r_z^2 + Q_{dd} d_z^2 + 2 z_z d_z Q_{xd} + 2 z_z r_z Q_{xr}$$
$$+ 2 r_z d_z Q_{dr} + z_{zz} Q_x + r_{zz} Q_r + d_{zz} Q_d$$
$$P_{gg} = Q_{xx} g_g^2 + Q_{rr} r_g^2 + Q_{dd} d_g^2 + 2 g_g d_g Q_{gd} + 2 g_g r_g Q_{gr}$$
$$+ 2 r_g d_g Q_{dg} + g_g g_g Q_x + r_{gg} Q_r + d_{gg} Q_d$$

where $x_g$, $x_t$, $x_z$ denote partial derivatives of the transform coordinates. Substitution of (3-5) into (3-1) gives a transformed wave equation for a source free region of the form:

$$Q_{xx} (x_g^2 + x_z^2 - \frac{1}{v^2} x_t^2 ) + Q_{rr} (r_g^2 + r_z^2 - \frac{1}{v^2} r_t^2 ) + Q_{dd} (d_g^2 + d_z^2 - \frac{1}{v^2} d_t^2 )$$
$$+ 2Q_{gd} (x_g d_g + x_z d_z - \frac{1}{v^2} x_t d_t ) + 2 Q_{gr} (x_g r_g + x_z r_z - \frac{1}{v^2} x_t r_t )$$
$$+ 2 Q_{rd} (r_g d_g + r_z d_z - \frac{1}{v^2} r_t d_t ) + Q_{x} (x_g + x_z - \frac{1}{v^2} x_t )$$
$$+ 2 Q_{xg} (r_g + r_z - \frac{1}{v^2} r_t ) + Q_{x} (d_g + d_z - \frac{1}{v^2} d_t ) = 0$$

(3-6)
Computing the required frame derivatives we have

\[ \frac{\partial r}{\partial (g,z,t)} = 0; 1; 0 \]  
\[ \text{(3-7a)} \]

\[ \frac{\partial d}{\partial (g,z,t)} = -\frac{2x}{v^2d} + \frac{1}{v} \left(1 + \frac{4x^2}{v^2d^2}\right)^{1/2} \]  
\[ \text{(3-7b)} \]

\[ \frac{\partial x}{\partial (g,z,t)} = \frac{1}{2} + \frac{r}{2v(d-r/v)} + \frac{2x^2r}{v^2d^2(d-r/v)} \]  
\[ \frac{x}{vd} ; -\frac{rx(1+4x^2/v^2d^2)^{1/2}}{vd(d-z/v)} \]  
\[ \text{(3-7c)} \]

For the second partials we have

\[ \frac{\partial^2 r}{\partial^2 (g,z,t)} = 0 ; 0 ; 0 \]  
\[ \text{(3-8a)} \]

\[ \frac{\partial^2 d}{\partial^2 (g,z,t)} = -\frac{\left(1+4x^2/v^2d^2\right)}{v^2(d-r/v)} ; 0 ; -\frac{4x^2}{v^2d^2(d-r/v)} \]  
\[ \text{(3-8b)} \]

\[ \frac{\partial^2 x}{\partial^2 (g,z,t)} = \frac{3rx}{v^3d(d-r/v)^2} + \frac{12x^3r}{d^3(d-r/v)^2v^5} ; 0 ; \]  
\[ + \frac{xr}{vd(d-r/v)^2} \left(2 + \frac{12x^2}{v^2d^2}\right) \]  
\[ \text{(3-8c)} \]

Substituting (3-7) and (3-8) into (3-6) gives
If we assume \( \tilde{v} = v = \text{constant} \) equation (3-9) becomes

\[
(1 + \frac{4x^2}{v^2d^2}) \frac{d^2}{(d-r/v)^2} \left\{ \frac{d^2}{4} + (\frac{r^2}{v^2} \frac{x^2}{v^2d^2})(1 - \frac{v^2}{\tilde{v}^2}) \right\} Q_{xx} + \frac{r}{v}(1 + \frac{4x^2}{v^2d^2}) \frac{2x}{vd} \frac{Q_{rd}}{v^2(d-r/v)} (1 - \frac{v^2}{\tilde{v}^2})
\]

\[
+ \frac{2x}{vd} Q_{xr} + \frac{2}{v} Q_{rd}
\]

\[
+ \frac{Q_{dd}}{(1 + \frac{4x^2}{v^2d^2})} \frac{1}{v^2d(d-r/v)^2} \left[ \frac{12x^2}{v^2d^2} - \frac{1}{v^2d(d-r/v)^2} \frac{12x^2}{v^2d^2} \right]
\]

\[
+ \frac{Q_d}{v^2(d-r/v)} \frac{1}{v^2d^2} = 0
\]

Equation (3-10) is the transformed wave equation which controls the behavior of moveout corrected wave fields. Since the only assumption made in transforming the wave equation was the constant velocity assumption, equation (3-10) models all possible wave effects regardless of their relevance to the problem of interest. With an eye toward deleting terms which deal with phenomena not central to our application, we will discuss some of the terms in equation (3-10).
Consider the first order terms $Q_x$ and $Q_d$. In the high frequency limit they are small compared to the remaining second order terms. We will delete both of these terms using the high frequency assumption that gradients of the wave field are more important than gradients of the coordinate system. While it is difficult to discern the wave phenomenon suppressed when the $Q_x$ term is neglected, the effects of neglecting the $Q_d$ term are fairly apparent. The $Q_d$ term increases the amplitude of the wave field by an amount proportional to inverse travel time, but it has little or no effect on phase. Neglecting $Q_d$, therefore, is nearly equivalent to neglecting amplitude changes due to geometrical spreading.

The final term we consider is the $Q_{rr}$ term. Unlike the first order terms, the importance of $Q_{rr}$ is dependent on the earth models one wishes to be able to handle. If the earth is plane layered we expect the moveout corrected wave fields recorded at the earth's surface to be the same as those recorded at depth. Thus, for a plane layered earth we have $Q_r = Q_{rr} = 0$. Recall that moveout correction approximately models the behavior of reflections even when the earth is only approximately layered. Because of this, moveout corrected data recorded over a nearly layered earth will be only moderately dependent on receiver depth. In this case, since $Q_r$ is small and $Q_r \gg Q_{rr}$, one might neglect $Q_{rr}$ in favor of $Q_r$. We shall adopt this small dip assumption and delete $Q_{rr}$ from our downward continuation equation. Deleting $Q_x$, $Q_d$ and $Q_{rr}$ we get

$$\frac{\partial}{\partial d} Q_{xr} + Q_{dr} = -\frac{v}{8} \frac{d^2}{(d-r/v)^2} \left( 1 + \frac{4x^2}{v_d^2} \right) Q_{xx}$$ (3-11)
Comparison of this development with Claerbout 1970 and 1971 shows that deletion of $Q_{rr}$ will yield an equation which models only upcoming waves. The earlier work also indicates that simply dropping $Q_{rr}$ will limit the range of accuracy of the resulting equation to dips of less than 15°. Claerbout 1971b gives economical procedures for extending the range of accuracy of the continuation equation to include reflector dips up to 45°. Claerbout and Johnson 1971 and also Riley 1975 give computer algorithms which can be used, with some modifications, to solve equations like (3-11).

Using methods similar to those shown here, Claerbout and Doherty 1972 derived an equation for downward continuing profiles. Expressed in the coordinates used in this thesis, their equation (equation 49) is

$$\frac{x}{d} Q_{xr} + Q_{dr} = -\frac{v}{8} \left( \frac{d}{d-r/v} \right)^2 Q_{xx}$$

(3-12)

The only difference between equations (3-11) and (3-12) is the lack of the $\frac{x^2}{v^2 d^2}$ dependence of the $Q_{xx}$ coefficient in (3-12). We find that the absence of this dependence in the published equation seriously degrades its performance for large offset data. In the earlier work it was felt that the $\frac{x}{d} Q_{xr}$ term might be important because its coefficient was first order in $x$. Here we find that downward continued wave fields are only weakly dependent on this term.

To put these comments on a firmer ground, we shall consider an example. As before, we shall use the point scatterer as our investigative probe since it is the most general reflector. Figure 3-4 indicates the recording geometry for the point scatterer synthetic profile we will use. Figure 3-4 shows ray paths only for the shallowest point in the synthetic data. Taking the angle between the actual ray paths and
Figure 3-4. The recording geometry for the data of figure 3-5. Actual ray paths are shown with solid lines. The ray paths assumed in the moveout correction are dotted. The paths shown are to the nearest and farthest receivers. The shortest shot-receiver offset is 8260', the longest is 16000'. Receiver spacing is 110'. The scatterer shown here causes the earliest arrivals shown in Figure 3-5.
the ray paths assumed in the moveout correction as a measure of dip, measurement on 3-4 shows that data recorded in that geometry fall within the 15° dip assumption made in dropping \( Q_{rr} \). The top left frame of figure 3-5 illustrates the surface synthetic profile data after they have been moveout corrected according to equations (3-2) (the effects of geometrical spreading and wavelet stretching due to moveout correction have been suppressed). To a viewer familiar with the section geometry, the most remarkable characteristic of this frame is that the data are not symmetric about the scatterer position. A look at the transformation equations (3-2) shows that symmetry should only be expected when the scatterer is directly under the source. The fact that this asymmetry exists should alert us to the possibility that insights gained from experience with sections may have to be modified somewhat if they are to be valid for profiles.

Frame 3-5b shows the same data after migration with the zero order terms (in x) of equation (3-11). That is with

\[
Q_{dr} = -\frac{v}{8} \left( \frac{d}{d-r/v} \right)^2 Q_{xx} \tag{3-13}
\]

An equivalent equation was used with much success on zero offset sections in the Claerbout-Doherty paper. Frame 3-5c shows the data migrated with the zero and second order terms from (3-11). That is with

\[
Q_{dr} = -\frac{v}{8} \left( 1 + \frac{4x^2}{v^2d^2} \right) \left( \frac{d}{d-r/v} \right)^2 Q_{xx} \tag{3-14}
\]

Frame 3-5d shows the data migrated with all the terms in (3-11).
Figure 3-5. Migrations of the profile data of figure 3-4. Frame a shows the unmigrated moveout corrected profile. The moveout correction velocity is 5000 ft/sec. Time ranges from 2.3 to 3.3 sec. The asymmetry of the data is due to the x dependence of the moveout
Fig. 3-5 Cont'd.

Correlation. Frame b shows the data of frame a after migration with equation (3-13). Frame c shows the same surface data after migration with equation (3-14). Frame d shows the same data after migration with equation (3-11). On the basis of these frames we can conclude that profiles can be migrated with good accuracy by an equation of the form

$$ Q_{dr} = -\frac{v}{8} \left(1 + \frac{4x^2}{v^2d^2}\right) \left(\frac{d}{d-r/v}\right)^2 Q_{xx} $$
Since the aim of migration and downward continuation is to obtain a moveout corrected profile which is a reflectivity map, the migrated data of figure (3-5) should resemble point scatterers. Because we are dealing with waves, we expect a focus rather than a point. In general, the size of this focus will depend both on the predominate wave length of the source wavelet and on the angular bandwidth of the initial conditions.

Obviously, frames (c) and (d) are much better approximations to focuses than (b). However, frames (c) and (d) are very similar. On the basis of this observation, we conclude that the $\frac{x^2}{v^2d^2}Q_{xx}$ term is required for migrating data of reasonable offset, while the $\frac{x}{d}Q_{xr}$ term is not. Note that the only noticeable effect of the $Q_{xr}$ term is an almost undetectable increase in the asymmetry of the foci. Since we know that any estimate of the reflectivity of a point scatterer should be symmetric about the point scatterer, inclusion of $Q_{xr}$ seems to move the data in the wrong direction. Part of the asymmetry of the migrated data is due to the asymmetric moveout function. However, this contribution is small since the foci are distributed over a narrow band in $x$ and the moveout is essentially constant across this band. Most of the asymmetry results from the biased nature of the angular bandwidth of the initial conditions. Although the receivers in the array are distributed equally along the $x$ axis, the angular bandwidth of the data is not. Since the angular bandwidth of the data is narrow for midpoints to the right of the scatterer, the focuses of 3-5 are wide on the right. The asymmetries in 3-5 occur because the appearance of the subsurface depends a great deal on the direction from which it is illuminated and the position from which it is viewed. The most familiar example of this kind of effect is the phases of the moon. Although illumination always comes
from the same location, the relative position of the observer changes during the month and thus the appearance of the moon also changes during the month.

Because a multi-offset section is constructed from a large number of profiles, we can make some predictions concerning section continuation on the basis of our work with profiles. We should expect that first order terms like $Q_x$ and $Q_d$ will be unimportant because they deal with low order effects like geometrical spreading. Also, we should find that directional derivative terms like $Q_{x_r}$ will control source-receiver location directivity effects. The result of their inclusion in a continuation equation will probably be a minor asymmetry in the migrated data.
Chapter 4 Downward Continuation of Sections

Introduction

In the previous chapter we developed equations useful for downward continuing data recorded as a single profile. As such, those equations were valid for wave fields generated by a single source at a fixed location. Here we wish to find equations useful for downward continuing data recorded as a section. That is, we want equations which can downward continue a wave field generated by many sources at varying locations.

An obvious approach might be to separately apply the equations of chapter 3 to the wave fields generated by each source. The main practical disadvantage of this approach is that data are often recorded with receiver cables which are too short to adequately define the initial conditions for the profile continuation equations. Initialization is particularly difficult in the common case where data are recorded with a moving array of a single source and receiver. Because of the high costs of overcoming this data initialization problem, we shall not use the profile approach to downward continue sections. Instead, we will reformulate the problem in a way in which data initialization difficulties are not so severe. In doing this, we shall use an approach like that found in chapter 3. First, we will formally describe the section data display in terms of a coordinate system. Then we will find transformation equations between these data display coordinates and the cartesian coordinates in which the wave equation is usually expressed. Finally, we will find a section continuation equation by transforming the wave equation into the section coordinates.
Coordinate Transformation and Wave Equation

As a first step in finding the section continuation equation, we must define both the field recording coordinates and the data display coordinates. As in the profile chapter we will assume that the reflectors are independent of one horizontal coordinate. Thus, our coordinate systems will describe horizontal distances only along the line of the section. Referring to Figure 4-1 we will fix the recording geometry. We chose to position the sources along the line of the section at locations \( s_1, s_2, \ldots, s_n \). The receivers (geophones) will be positioned along the same line at locations \( g_1, g_2, \ldots, g_n \). We will use \( t \) for reflection travel time and \( z \) for depth (+z down). The section data display coordinates will be defined as follows: \( h \) is half the surface source-receiver offset; \( y \) is the horizontal coordinate of the surface source-receiver midpoint measured from a fixed origin; \( d \) is a moveout corrected two way travel time and \( r \) is the receiver depth (+r down).

Figure 4-1. Geometry for the offset midpoint coordinate system.
The equations which describe the transformation from the field recording coordinates \((s, g, t, z)\) to the section coordinates \((y, h, d, r)\) are given by:

\begin{align*}
h &= \frac{(g-s)}{2} + z \frac{(g-s)}{2v} / \left( t^2 - \frac{(g-s)^2}{v^2} \right)^{1/2} \quad (4-1a) \\
y &= \frac{(g+s)}{2} + z \frac{(g-s)}{2v} / \left( t^2 - \frac{(g-s)^2}{v^2} \right)^{1/2} \quad (4-1b) \\
d &= \left( t^2 - \frac{(g-s)^2}{v^2} \right)^{1/2} + \frac{z}{v} \quad (4-1c) \\
r &= z \quad (4-1d)
\end{align*}

We have used \(v\) as the constant moveout correction velocity. As in the profile frame we have included a depth dependence so that \(h, y,\) and \(d\) are constant along a plane layer reflection ray path. There is no coordinate describing the shot depth since we are explicitly requiring that sources remain at the earth's surface, \(z=0\).

Equations 4-1 are of course invertible. Their inverse is given by

\begin{align*}
g &= y + h - \frac{2hr}{vd} \quad (4-2a) \\
s &= y - h \quad (4-2b) \\
t &= \left( d^2 + 4h^2 / v^2 \right)^{1/2} \frac{(d - r)}{d} \quad (4-2c) \\
z &= r \quad (4-2d)
\end{align*}
Each wave field generated by each individual source must separately satisfy the wave equation. Thus, instead of one wave equation, the section problem requires \( n \) wave equations if there are \( n \) sources. In terms of the field coordinates the equations which govern the wave field are

\[
\frac{1}{v^2} p_{zz} - \frac{1}{v^2} p_{tt} = \delta(s_i - g, t, z) \quad i = 1, n \quad (4-3)
\]

As in chapter 3 we are using a 2-D scalar wave equation. We are also using subscripts to denote derivatives, \( \tilde{v} \) to denote the variable wave velocity and the delta function to denote the sources. As a convenience, we will assume that there are a continuum of sources located along the line of the section. If we do this, equation (4-3) can be considered a single equation in four variables \( (g, s, t \text{ and } z) \).

Since we are not interested in describing wave fields at the source locations, we can drop the delta function from (4-3). Doing this we are left with the equation to be transformed into the section coordinates

\[
\frac{1}{v^2} p_{gg} + \frac{1}{v^2} p_{zz} = 0 \quad (4-4)
\]

**Continuation Equation**

Having equations (4-1), (4-2) and (4-4) we can find the continuation equation for \( Q(y,h,d,r) \), the wave field in the section coordinate system. In doing this we will adopt the high frequency assumption and the small dip assumption discussed in chapter 3. Using these assumptions and
the chain rule, the continuation equation for $Q$ is 

$$
( h_{g}^2 + h_{z}^2 - \frac{1}{v^2} h_{t}^2 ) Q_{hh} + ( y_{g}^2 + y_{z}^2 - \frac{1}{v^2} y_{t}^2 ) Q_{yy}
$$

$$
+ Q_{dd} ( d_{g}^2 + d_{z}^2 - \frac{1}{v^2} d_{t}^2 ) + 2( h_{g} y_{g} + h_{z} y_{z} - \frac{1}{v^2} h_{t} y_{t} ) Q_{hy}
$$

$$
+ 2( h_{g} d_{g} + h_{z} d_{z} - \frac{1}{v^2} h_{t} d_{t} ) Q_{hd} + 2( h_{g} r_{g} + h_{z} r_{z} - \frac{1}{v^2} h_{t} r_{t} ) Q_{hr}
$$

$$
+ 2( y_{g} d_{g} + y_{z} d_{z} - \frac{1}{v^2} y_{t} d_{t} ) Q_{yd} + 2( y_{g} r_{g} + y_{z} r_{z} - \frac{1}{v^2} y_{t} r_{t} ) Q_{yr}
$$

$$
+ 2( d_{g} r_{g} + d_{z} r_{z} - \frac{1}{v^2} d_{t} r_{t} ) Q_{dr} = 0
$$

(4-5)

where $h_{z}$, $h_{g}$, $h_{t}$ etc. denote partial derivatives of the section coordinates.

We shall need expressions for the partial derivatives in equation (4-5). Thankfully, there is an immediate simplification. The definitions of $y$ and $h$ differ only by the sign of $s$ in the first terms of (4-1a) and (4-1b). Thus, we have

$$
\frac{\partial y}{\partial (g,t,z)} = \frac{\partial h}{\partial (g,t,z)}
$$

(4-6)

The expressions for the remaining derivatives are

$$
\frac{\partial r}{\partial (t,z,g)} = 0, 1, 0
$$

(4-7a)

$$
\frac{\partial d}{\partial (t,z,g)} = \alpha, \frac{1}{v}, \frac{-2h}{v^2 d}
$$

(4-7b)

$$
\frac{\partial y}{\partial (t,z,g)} = \frac{-rh}{vd} \alpha, \frac{h}{dv}, \frac{1}{2} + \frac{r}{2v\beta} + \frac{2rh^2}{v^3 d^2 \beta}
$$

(4-7c)

where $\alpha = (1 + \frac{4h^2}{v^2 d^2})^{1/2}$ and $\beta = (d - r/v)$

(4-8)

Using (4-6), (4-7) and (4-8) equation (4-5) becomes
\[
\frac{2}{v} Q_{dr} + \frac{2h}{vd} (Q_{yr} + Q_{hr}) + \frac{\alpha^2}{\beta^2} \left\{ \frac{d^2}{4} + \frac{r^2h^2}{v^4d^2} \gamma \right\} (Q_{yy} + 2Q_{yh} + Q_{hh})
\]

\[- \frac{2hra^2\gamma}{v^3d} (Q_{dh} + Q_{dy}) + \frac{\alpha^2}{v^2} \gamma Q_{dd} = 0 \quad (4-9)\]

where \(\alpha\) and \(\beta\) are as defined in (4-8) and

\[
\gamma = (1 - \frac{v^2}{V^2}) \quad (4-10)
\]

If the moveout velocity is the same as the velocity in the wave equation, then \(\gamma = 0\) and equation (4-9) becomes

\[
\frac{2}{v} Q_{dr} + \frac{2h}{vd} (Q_{yr} + Q_{hr}) + \frac{d^2\alpha^2}{4 \beta^2} (Q_{yy} + 2Q_{yh} + Q_{hh}) = 0 \quad (4-11)
\]

Substitution for \(\alpha\) and \(\beta\) and rearrangement gives

\[
Q_{dr} + \frac{h}{d} (Q_{yr} + Q_{hr}) = -(\frac{d}{d-r/v})^2 \frac{v}{8} \left( 1 + \frac{4h^2}{v^2d^2} \right) (Q_{yy} + 2Q_{yh} + Q_{hh}) \quad (4-12)
\]

Equation (4-12) governs the downward continuation of multi-offset sections (sections of CMP gathers). Although we could devise a scheme for numerically integrating (4-12) and use it to perform downward continuation, we would rather work with a simpler equation if possible. Much of the complexity of (4-12) is due to the \(Q_{hr}\), \(Q_{yh}\) and \(Q_{hh}\) terms. The presence of these terms in the continuation equation seems to indicate that common offset sections cannot be migrated separately. This implied coupling not only increases the computer time and storage required to migrate a single common offset section, it also precludes the possibility of migrating data recorded with a moving array of a single source and receiver. In an effort to simplify (4-12) we will examine these and other terms to see if they are important in describing the
behavior of the data we wish to model.

Recall that equation (4-12) governs properly moveout corrected data. Moveout correction approximately models travel time variations due to shot-receiver offset even when the reflectors are sub-horizontal. Because of this, when reflector dip is not large, $Q$ should be only weakly dependent on $h$. Thus, for moderate dip and offset we should have

$$Q_y > Q_h \quad \text{or} \quad \frac{Q_h}{Q_y} << 1 \quad (4-13)$$

If inequality (4-13) is strong enough we can use it to justify deleting the coupling terms from (4-12) and thus allow separate migration of common offset sections.

To get an idea of the strength of inequality (4-13) we shall need some results from chapter 5. In chapter 5 we demonstrate that the rms velocity of a reflection from a dipping plane is given by

$$v_{\text{rms}} = \tilde{v}/\cos\phi \quad , \quad (4-14)$$

where $\phi$ is the dip of the reflector. The travel times of such reflections are given by

$$t^2(h) = t_0^2 + \frac{4h^2}{v_{\text{rms}}} \quad , \quad (4-15)$$

where $t_0$ is the zero offset travel time. After moveout correction with the true velocity these travel times are

$$t^2(h) = t_0^2 + 4h^2 \left( \frac{1}{2} - \frac{1}{v_{\text{rms}}} \right) \quad (4-16)$$

Using a binomial expansion and (4-14), equation (4-16) becomes
\[ \tau = t_0 - \frac{2h^2}{v^2 t_0} \sin^2 \phi \]  
(4-17)

Taking a derivative with respect to \( h \) we get

\[ \tau_h = \frac{\partial \tau}{\partial h} = - \frac{4h}{v^2 t_0} \sin^2 \phi \]  
(4-18)

Simple geometry shows that for a zero offset section we have

\[ \frac{\partial \tau}{\partial y} = \tau_y = \frac{2 \sin \phi}{v} \]  
(4-19)

If we disregard the fact that the apparent dip of a reflector is a weak function of offset, and use a plane wave assumption we have

\[ \frac{Q_h}{Q_y} \approx \frac{\tau_h}{\tau_y} \]  
(4-20)

Substituting from (4-18) and (4-19), equation (4-20) becomes

\[ \frac{Q_h}{Q_y} \approx - \frac{2h \sin \phi}{v t_0} = - \sin \phi \tan \theta \]  
(4-21)

where \( \theta \) is as defined in Figure 4-1.

We can use (4-21) as a rough guide to the importance of the \( Q_h \) terms in equation (4-12). Table 4-1 shows equation (4-21) evaluated for particular values of \( \theta \) and \( \phi \). From the table we can conclude that \( Q_h \) is neglectable compared to \( Q_y \) for many commonly encountered dips and offsets. (The calculations in the figures of chapter 5 indicate that the errors implied by the Table 4-1 are probably larger than the errors which actually occur.) The table also indicates that the terms in (4-12) containing first order \( h \) derivatives cannot be neglected relative to terms containing first order \( y \) derivatives if either dip or offset is not small.
TABLE 4-1. Ratios of terms in equation (4-12) for various dips and emergence angles. Columns A and B are based on equation (4-21). Column C is the result of numerical experiments on surface synthetic data. The precision of column C is about + 20%. Because correctly migrated data are independent of offset, the average value of these ratios during migration should be about half the size shown in the table.

| θ   | φ   | $\frac{|Q_{h}|}{Q_y}$ | $\frac{|Q_{hh}|}{Q_{yy}}$ | $\frac{|Q_{hh}|}{Q_{yy}}$ |
|-----|-----|----------------------|-------------------------|-------------------------|
| 30° | 30° | .29                  | .08                     | .09                     |
| 45° | 15° | .26                  | .07                     | .09                     |
| 30° | 15° | .15                  | .02                     | .03                     |
| 15° | 15° | .07                  | .005                    | .01                     |

A  B  C

To get a handle on the function of the non-neglectable $Q_{hr}$, $Q_{yr}$ and $Q_{yh}$ terms, we will investigate their effect on the migration of synthetic data that would be recorded over a point scatterer located at $y=y_0$. Such surface data are symmetric about $y=y_0$ and $h=0$. Consider first the directional derivative terms $Q_{yr}$ and $Q_{hr}$. Inequality (4-13) implies that the $Q_{yr}$ term should be more important than the $Q_{hr}$ term. On the basis of our study of profiles in chapter 3, we expect that the main result of the inclusion of $Q_{yr}$ in a continuation equation will be the introduction of a minor asymmetry in the focus of migrated scatterer data.
The migrated data of Figure 4-2 were constructed with an equation of the form

\[ \frac{Q_{dr}}{d} + \frac{h}{d} Q_{yr} = -\frac{v}{8} (1 + \frac{4h^2}{v^2d^2}) \left(\frac{d}{d-r/v}\right)^2 Q_{yy} \]  

(4-22)

As expected they show a small amount of skewing due to the presence of \( Q_{yr} \) in (4-22). As in chapter 3 we interpret this skewing to be the result of source-receiver directivity effects.

Next consider the \( Q_{yh} \) term. Because the dip (in \( y \)) of the point scatterer hyperbolic changes sign at \( y=y_0 \), \( Q_y \) must also change sign at this location. Equation (4-14) shows that apparent moveout velocity is independent of the sign of dip. Thus, the residual moveout of the point scatterer data and \( Q_h \) must be symmetric about \( y=y_0 \). Because of this we can conclude that \( Q_{yh} \) will change sign at \( y=y_0 \) and thus, must cause some skewing of downward continued scatterer data. Figure 4-3 shows data migrated with and without the \( Q_{yh} \) and \( Q_{yr} \) terms of (4-12). As expected, the inclusion of these terms in the continuation equation results in some asymmetry in the migrated data. Note, however, that the position of the focus is apparently unchanged by their presence and that the data continued without \( Q_{yh} \) and \( Q_{yr} \) appear properly migrated.

Although Table 1 indicates that the \( Q_{yr} \), \( Q_{hr} \) and \( Q_{yh} \) terms cannot be deleted from (4-12) on the basis of size, Figures 4-2 and 4-3 indicate that the effects of their deletion are not large. The same figures also indicate that migrated data generated with equations including these terms appear to exhibit some skewing or
Figure 4-2.  Scatterer data illustrating the effects of including the $Q_{yr}$ term in a continuation equation.

Frame A shows a moveout corrected large offset section (40° rays) that would be recorded over a point scatterer. Frame B shows the data migrated without the $Q_{yr}$ term using equation (4-23). Frame C shows data migrated with

$$Q_{dr} \frac{h}{d} Q_{yr} = -\frac{v}{8} \left( \frac{d}{d-r/v} \right)^2 \left( 1 + 4h^2/v^2d^2 \right) Q_{yy}.$$

The major effect of the inclusion of the $Q_{yr}$ term is the generation of the slight asymmetry seen in frame C.
Figure 4-3. Scatterer data illustrating the effects of the inclusion of the $Q_{yr}$ and $Q_{yh}$ terms in a continuation equation. The offsets for frames A, B, C are 700', 2800', and 4900' respectively. The corresponding ray angles are 7°, 25° and 39°. Each frame is divided by white bands into 3 regions. The top regions illustrate moveout corrected common offset sections constructed from surface data. The middle region illustrates the results of migrating the top region with equation (4-23). The lower regions show the surface data migrated with an equation of the form

$$Q_{dr} + \frac{h}{d} Q_{yr} = - \left( \frac{d}{d-r/v} \right)^2 \frac{v}{8} \left( 1 + \frac{4h^2}{v^2d^2} \right) (Q_{yy} + 2Q_{yh})$$
asymmetry which one would not usually expect in earth models. Unfortunately, our present level of understanding and observational experience are not sufficient to allow us to make definitive statements about the exact role of these terms in wave equation migration. Perhaps they arise solely because we downward continue receivers but not sources. (Ideally we would like to downward continue both sources and receivers simultaneously.) Possibly, better migrations could be achieved with equations containing $Q_{yr}$, $Q_{hy}$ and $Q_{yh}$ if we were to use a reflector mapping principle that accommodated source-receiver directivity effects.

Regrettably, in this thesis we must leave the question of the role of $Q_{yh}$, $Q_{yr}$ and $Q_{hr}$ unanswered. Fortunately, we can still make use of the equations we have derived, since all our calculations show that, for models fitting the assumptions made in chapters 1 and 3, the neglection of these terms, at worst, causes only travel time changes which are very much smaller than a wave period. Because errors of this size are virtually undetectable on field data, we can neglect them leaving an equation of the form:

$$Q_{dr} = -\frac{v}{8} \left( 1 + \frac{4h^2}{v^2d^2} \right) \left( \frac{d}{d-r/v} \right)^2 Q_{yy}.$$

One might assume that because of the deletion of $Q_{h}$ terms, equations like (4-23) cannot model the interaction of dip in the $y$ direction with curvature in the $h$ direction. Specifically, one might think that (4-23) does not model the dip dependence of apparent moveout velocity described in chapters 2 and 5. Surprisingly, this is not true. Many of the figures in chapter 5 demonstrate that equation (4-23) accurately models this phenomena. In fact, equation (4-23) is the continuation operator we shall use in chapter 5 to remove structural effects from the data prior to velocity estimation.
Chapter 5. Velocity Estimation

Introduction

In chapter 1 we noted that because most conventional velocity estimation techniques assume horizontal reflectors their performance tends to degrade when the reflectors are curved, dipping or discontinuous. In this chapter we make a more detailed study of this degradation. On the basis of this study, we conclude that velocity estimates may be improved if they are based on downward continued data rather than on surface data. Using two synthetic examples we show both that velocity estimates can be improved by downward continuation and that this improvement does not depend critically on the velocity used in the continuation equation. Next we discuss how downward continuation might be used to allow accurate velocity estimates to be made from data recorded in areas where the reflectors have little lateral continuity. As a finale we illustrate the effects of downward continuation on velocity estimates made from some Gulf Coast field data.

Effects of Reflector Structure on Velocity Estimates

Levin (1971) showed that, when viewed on a common midpoint gather, the arrival times of reflections from dipping planes follow hyperbolic trajectories. Levin also showed that the rms velocities, \( v_{\text{rms}} \), obtained from such data are always greater than the true wave velocity \( \hat{v} \). Specifically, the relationship is

\[
v_{\text{rms}} = \frac{\hat{v}}{\cos(\phi)}
\]

where \( \phi \) is the dip of the reflector and \( \hat{v} \) is the true velocity.

Figures 5-1 and 5-2 illustrate this dip dependence of velocity which we shall call the Levin effect. Frame 5-1B shows moveout corrected gathers which would be recorded over a point scatterer. Residual moveout
Figure 5-1. Point scatterer data illustrating the Levin effect and velocity diffusion. Frame A shows a zero offset section constructed from surface data that would be recorded over a point scatterer. Frame B shows a set of moveout corrected gathers constructed from all offsets of the data in frame A. The inset in frame B depicts the axes used in that frame. The position of the large offset trace in each gather of frame B corresponds to the position of that gather's common midpoint in frame A. The lines and brackets connecting the two frames illustrate this correspondence for two of the gathers. Because the gathers have been moveout corrected with the true velocity, any curvature seen on them is due to Levin effect residual moveout. The presence of residual moveout on a particular gather indicates that hyperbola fitting velocity estimation techniques will make incorrect velocity estimates from that gather.
caused by the Levin effect is readily apparent on these data. The velocity estimates which would be made from the data of Figure 5-1 are shown in Figure 5-2. The coherence of each common midpoint gather along the various hyperbolic trajectories \( \tau(d_0, h, v_{\text{rms}}) \) is displayed on the midpoint versus time plot of Figure 5-2. The hyperbolic trajectories, \( \tau(d_0, h, v_{\text{rms}}) \) are given by

\[
\tau(d_0, h, v_{\text{rms}}) = (d_0^2 + \frac{4h^2}{v_{\text{rms}}^2})^{1/2}
\]

where \( d_0 \) is a zero offset travel time. The coherence measure, \( c(y, v_{\text{rms}}) \), is a partially normalized sum of the data along each trajectory, given by

\[
c(y, v_{\text{rms}}) = \frac{\sum_{h=0}^{h_{\text{max}}} \left( \sum_{d_1}^{d_2} Q(y, \tau) \right)^2}{\sum_{h=0}^{h_{\text{max}}} \left( \sum_{d_1}^{d_2} Q(y, \tau) \right)^2}^{1/2}
\]

For Figure 5-2 the outer time gate, \((d_2 - d_1)\), included all of the data in Figure 5-1. Peak values of \( c(y, v_{\text{rms}}) \) indicate the best estimate of velocity for each midpoint. As predicted by equation (5-1), velocity error is greatest where the apparent dip of the data is largest.

Often equation (5-1) is the only correction needed to obtain true velocities from the rms velocities of reflections from curved or dipping interfaces. To make this correction, dip must be estimated directly from the data. Usually, this is not difficult. Problems arise when there are conflicting dips or when there is rapid horizontal dip variation. Diffracted events are also a problem since their apparent dip as measured on a section is not necessarily the angle needed for equation (5-1). Figure 5-3 illustrates problems which can be encountered with point scatterer diffractions.
Figure 5-2. Velocity estimates for the data of Figure 5-1. Displayed on the velocity versus midpoint plot, is a partially normalized sum of the CMP gather data along hyperbolic trajectories corresponding to various rms velocities. Peak values indicate the best estimate of velocity for each midpoint. True velocity is 5000 ft/sec. Estimated velocities are highest where the apparent dip of the data is greatest. Velocity estimates are correct near the scatterer where the dip of the data is nearly zero.
Figure 5-3. Apparent dip of diffraction events cannot be used to correct moveout velocities for the effects of structure. At midpoints far from the scatterer, the apparent dip of the diffraction is independent of shot and receiver location. The dip is governed solely by the asymptote of the hyperbola. For these midpoints the ray paths for all offsets are nearly horizontal. Because of this, the travel times of the reflections are nearly independent of offset and the moveout velocity is very large. As the midpoint goes to infinity, travel time becomes independent of offset and the moveout velocity becomes infinite. In this case equation (5-1) requires the correction angle to be 90°. Clearly the apparent dip of the data cannot be 90°, since that would imply that the material velocity was zero. The angle required to correct the moveout velocities is shown in the figure as $\phi$. Note that $\phi$ becomes 90° for midpoints far from the scatterer. Since measurement of $\phi$ requires knowledge of the scatterer position, we can conclude that dip estimation for diffracted data cannot be a local process.
Reflections from a point scatterer illustrate a second characteristic of non-planar data which can cause difficulties for velocity estimators. The hyperbolic arrival times of point scatterer reflections tend to diffuse velocity information away from the scatterer location. Even if accurate dip corrections are made, erroneous velocity estimates may be obtained from these data because the location of the estimate may not coincide with the reflection point of the waves upon which the estimate is based. Scatterer data illustrates the general principle that reflections from non-horizontal reflectors, tend to diffuse velocity information in both space and time. Since the dip angles encountered in reflection seismology are generally less than 45°, velocity information tends to diffuse more horizontally than vertically. Thus, diffusion is most important when velocity is laterally variable. However, even in the laterally invariant velocity models treated in this thesis, velocity diffusion may result in contamination of deep velocity estimates with estimates based on reflections from shallow non-horizontal reflectors.

Preprocessing with Downward Continuation

The incorrect positioning of velocity estimates which we have called velocity diffusion occurs when the location of a portion of data on a seismic section does not coincide with the location of its reflection point. In earlier chapters we found that downward continuation can be used to position or migrate all data on a section to their reflection points. We can exploit this property of downward continuation to improve velocity estimates by using downward continuation as a preprocessor for conventional velocity estimation techniques. If we do this, the resultant velocity estimates should not exhibit any velocity diffusion effects.
This approach has an additional advantage: it suppresses errors due to residual moveout caused by the Levin effect. To see why this occurs, consider data recorded over a point scatterer. Velocity estimates made from such surface data must be dip corrected as indicated in equation (5-1) and Figure 5-3. Migration collapses point scatterer hyperbolics to focuses for which dips measured as shown in Figure 5-3 are zero. Because of this, equation (5-1) implies that no dip corrections are necessary for velocity estimates based on downward continued scatterer data. Since the wave equation is linear, we can conclude that no dip corrections are necessary for any reflector geometry, on the basis of this point scatterer example.

Figures 5-4 through 5-7 demonstrate the usefulness of downward continuation in removing diffusion and Levin effect phenomena from velocity estimates. Figure 5-4 shows the earth model used to generate the data for 5-5, 5-6 and 5-7. The leftmost frames of 5-5 show two moveout corrected common offset sections generated by using the data of 5-4 as initial conditions in the time reversed version of equation (4-23). Differences between the near trace section and the far trace section are due to structurally caused residual moveout. The center frames of 5-5 show the same surface data after migration. The rightmost frames show the data after migration with 10% too low of velocity. In spite of the erroneous velocity, migration has removed most diffraction effects from these data.

Figure 5-6 shows moveout corrected gathers constructed from the data of Figure 5-5. The surface gathers exhibit much structurally caused residual moveout. The migrated gathers, on the other hand, are independent of offset. The undermigrated gathers on the far right of Figure 5-6
Figure 5-4. Earth model used to generate the synthetic data shown in Figures 5-5 through 5-7. Time ranges from 2.5 to 3.5 sec. Trace spacing is 55 feet. The largest dips in the bowl are between 15° and 20°. The arrows indicate the position of the velocity estimates shown in Figure 5-7.
Figure 5-5. Surface data and earth reconstructions for the earth model of Figure 5-4. The leftmost
Figure 5-5 cont'd.
frames are moveout corrected surface data sections constructed by using the earth model of 5-4 as initial conditions in the time reversed version of equation (4-23). Water velocity was used for both the moveout and migration velocity. The upper frames are small offset data (h=600'). The lower frames are large offset data (h=3600'). The center frames show the surface data after migration. Reconstruction is not exact due to dip filtering used in generating the surface data. The rightmost frames show the same surface data after migration with a velocity which was 10% too small. In spite of the incorrect velocity, most of the diffractions apparent in the surface data have been removed by downward continuation. These frames illustrate the fact that migration quality is moderately insensitive to velocity error if reflector dips are not large.
Figure 5-6. Moveout corrected common midpoint gathers for the data of Figure 5-5. The axes for these plots are the same as those for frame B of Figure 5-1. Frame A shows the gathers constructed from the surface data. Curve on the surface data is due to structurally caused residual moveout. The largest dips correspond to a velocity error of about 4%. Frame B shows the gathers constructed from the properly migrated data of frame 5-5. The data in this frame are virtually independent of offset. Since downward continuation has removed velocity diffusion and Levin effect phenomena, correct velocities can easily be estimated from these data. Frame C displays the gathers constructed from the undermigrated data of figure 5-5. Downward continuation has removed most of the structurally caused residual moveout and velocity diffusion. Because of this, velocity estimates based on frame C should be superior to those based on frame A.
illustrate that downward continuation, even with an erroneous velocity, can result in dramatic reductions in velocity diffusion and Levin effect phenomena.

Figure 5-7 depicts the velocity estimates made from two common midpoint gathers of the data of Figure 5-5. The locations of the gathers are given by the arrows in Figure 5-4. Velocity estimates based on surface data are shown on the left of 5-7. Estimates based on migrated data are shown on the right. Velocity diffusion causes the number of 'events' on the surface estimates to be larger than the number on the migrated estimates. Erroneous velocity estimates caused by the Levin effect are apparent on the surface estimates.

The migrated data for Figures 5-4 through 5-7 were constructed with equation (4-23). Thus, each common offset section was migrated separately, using a constant moveout and continuation velocity. The moveout and continuation velocities were the same as that used to generate the surface initial conditions. Because of this, these figures demonstrate only that downward continuation with the true velocity allows accurate estimation of that already known velocity.

Downward Continuation with Erroneous Velocities

When downward continuation is used in a velocity estimation scheme, the choice of a continuation velocity must be made before the true velocities are determined. This order of operations almost guarantees that the continuation velocities used in velocity estimation will differ from the true velocities. In the previous section we demonstrated that continuation with the correct velocity, results in improved estimates. Here we attempt to analyze the results which can be expected in the realistic case where the continuation velocity is incorrect.
Figure 5-7. Velocity estimates from the data of Figure 5-5. The estimates on the left are based on surface data. The estimates on the right are based on properly migrated data. The positions of midpoints 18 and 30 are shown in Figure 5-4. The coherence measure displayed on the plots is the same as that used in 5-1. The estimate shown for a particular time represents an average over a 50 millisecond time gate centered on that time. True velocity is 5000 ft/sec. The velocity estimated from the downward continued data is essentially the true velocity. The surface data plot shows estimates as high as 5300 ft/sec.
To do this we need to consider equation (4-9) from the previous chapter. Copying this equation from chapter 4 we have

\[ Q_{dr} + \frac{h}{d} (\partial_y + \partial_h) Q_r = -\frac{va^2}{2\beta^2} \left( \frac{d^2}{4} + \frac{r\gamma^2}{v_d^2} \right) (\partial_y + \partial_h)^2 Q \]

\[ + \frac{h \omega^2 \gamma}{v_d^2 \beta} (\partial_y + \partial_h) Q_d - \frac{a^2}{2w} \gamma Q_{dd} \]  \hspace{1cm} (5-4)

where

\[ a^2 = 1 + \frac{4h^2}{v_d^2} \quad ; \quad \beta = d - r/v \quad \text{and} \quad \gamma = (1 - \frac{v^2}{\tilde{v}^2}) . \] \hspace{1cm} (5-5)

Equation (5-4) is a continuation equation which is valid for a constant moveout velocity, \( v \), but a variable wave velocity \( \tilde{v} \). The \( \gamma \) dependent terms of (5-4) turn on only when the moveout correction velocity differs from the true velocity. Thus, one way of analyzing the errors that may result from continuation with an erroneous velocity, is to examine the \( \gamma \) dependent terms of equation (5-4). We shall use that approach here. In analyzing (5-4) we will neglect the effects of amplitude variation due to geometrical spreading and of wavelet stretching due to moveout correction. In making the analysis, we will examine each type of term in (5-4) in succession. If a term is found to be unimportant under some reasonable assumption, we will drop it from the continuation equation. After making all reasonable deletions, we will compare the resulting equation to (4-23), the equation used earlier to remove structural effects when velocity was known. If these two equations are markedly different we can expect some problems to result from downward continuation with (4-23).

As a first step in the analysis we will drop the \( Q_{hr}, Q_{yr} \) and \( Q_{yh} \) terms on the basis of the arguments given in chapter 4. Next
we study the behavior of the remaining terms of (5-4) under the assumption that the earth's velocity structure is layered and that the reflectors have no dip. In this case $Q_y = 0$ and terms depending on $Q_y$ may be dropped from (5-4) leaving

$$Q_{dr} = -\frac{v_0^2}{2\beta^2} \left( \frac{d^2}{4} + \frac{r^2 h^2}{v^2 d^2} \right) Q_{hh} + \frac{hr\alpha^2}{v^2 \beta} Q_{hd}$$

$$- \frac{\gamma}{2v} \left( 1 + 4h^2 / v^2 d^2 \right) Q_{dd}$$

Although equation (5-6) looks complicated what it does is not complicated. If $\gamma=0$, the design of the coordinate system requires that surface and buried receiver data be equivalent. Thus, when $v=v_0$ and $Q_y=0$ continuation with (5-6) is a null operation. When $\gamma\neq0$ the situation is a bit different.

Moveout correction of surface data requires correction for both the upgoing and downgoing legs of the reflection path. Buried receiver data requires correction only for the downgoing path. Residual moveout caused by differences between the true velocity and the moveout correction velocity is associated with both legs of a reflection path. Therefore, when $\gamma\neq0$ buried receiver data have half the residual moveout of surface receiver data (here we are considering the travel times of the zero offset arrival to be part of the residual moveout).

In the case we are studying, this reduced moveout is the main difference between surface and buried receiver data. Accordingly, when there is no dip and $\gamma\neq0$, the main result of continuation with (5-6) is a 50% reduction of residual moveout.
If the residual moveout of an event is small compared to its zero offset travel time, one can show that the time shifting term of equation (5-6), $Q_{dd}$, performs virtually all of the required moveout reduction. Thus, when velocity error and offset are moderate, the terms depending on $Q_{hh}$ and $Q_h$ in (5-6) do not significantly affect the downward continued data and they may be neglected. If downward continuation is used as part of a velocity estimation procedure it may not be desirable to model behavior governed by any of the $h$ dependent terms of (5-6), including $Q_{dd}$. This is because there is a great practical advantage to be gained by insuring that residual moveout due to non-zero values of $\gamma$ be independent of receiver depth. If this is done, most velocity estimation programs designed for use on surface data can be applied to downward continued data without modification. It is for this reason that in velocity estimation applications we will drop all the $h$ dependent terms in (5-6), giving a continuation equation of the form

$$Q_{dr} = -\frac{v}{2} \frac{\alpha^2}{\beta^2} \left( \frac{d^2}{4} + \frac{r_{hh}^2 \gamma}{v^4 d^2} \right) Q_{yy} + \frac{hr\alpha^2 \gamma}{v^2 d \beta} Q_{dy} - \frac{\gamma}{2v} Q_{dd} \quad (5-7)$$

By choosing to ignore the $h$ dependent terms of (5-6) we have lost the ability to accurately model situations where velocity varies horizontally. More precisely, we have made the assumption that velocity does not vary significantly over distances comparable to the receiver cable length. Since this is the assumption made when velocity is estimated on the basis of correlations of data along hyperbolic paths, we feel that it is justified here. This assumption along with the fact that our continuation equations do not downward continue sources, is the main reason that the methods given in this thesis are strictly valid only for the first class earth models discussed in chapter 1.
The final term in (5-7) we shall consider is the $Q_{dy}$ term. As we noted above, when $y \neq 0$ correctly downward continued data have residual moveout, including an incorrect zero offset travel time. A look at the definition of $y$ in equation (4-1b) shows that $y$ depends not only on the source and receiver coordinates, $(g,s,z)$, but also on $v$ and $t$. This dependence along with the residual moveout causes each offset of the correctly migrated data corresponding to the same reflection point to appear at a different position on the $y$ axis. This effect is undesirable because it means that migrated common midpoint gathers are not common reflection point gathers. Integration over $d$ using a high frequency assumption shows that $Q_{dy}$ is a horizontal shifting term. Its predominate function is to shift, without change of form, each offset of the data a different distance along the $y$ axis. Because of this, we conclude that $Q_{dy}$ models the undesirable effect discussed above. Since we don't need to model this effect we will delete the $Q_{dy}$ term from (5-7) leaving

$$Q_{dr} = -\frac{v_0^2}{2\beta^2} \left( \frac{d^2}{4} + \frac{r^2h^2y^2}{v^2d^2} \right) Q_{yy} - \frac{2y}{v} Q_{dd} \quad (5-8)$$

Notice that we could have used this same reasoning to delete the other horizontal shift term, $Q_{hd}$, had it not been neglected earlier.

Equation (5-8) is the equation we will compare to (4-23) to get an idea of how large migration velocity error must be before downward continuation causes degradation of velocity estimates. Copying equation (4-23) from chapter 4 we have

$$Q_{dr} = -\frac{v}{8} \left( \frac{d}{d-r/v} \right)^2 (1 + 4h^2/v^2d^2) Q_{yy} \quad (5-9)$$
If $\gamma$ is such that (5-8) differs markedly from (5-9) we should expect some problems to develop with the downward continuation.

Consider first, the coefficient of $Q_{yy}$ in (5-8). Substituting for $\alpha^2$ and $\beta^2$ and rearranging we have

$$-\frac{v}{2} \frac{\alpha^2}{\beta^2} \left( \frac{d^2}{4} + \frac{r^2 h^2 \gamma}{v d^2} \right) = -\frac{v}{8} \left(1 + \frac{4h^2}{v^2 d^2} \right) \left( \frac{d}{d-r/v} \right)^2 \left(1 + \frac{4r^2 h^2 \gamma}{v d^4} \right)$$ (5-10)

The over-barred term in equation (5-10) represents a difference between (5-8) and (5-9). For reasonable values of $h$ and $\gamma$ this term is small and its absence from (5-9) will not cause much error. For $v = 1.2 \bar{v}$ and $45^\circ$ emergence angle ($\theta$ in Figure 4-1) its average value is .03.

Next consider the time shifting term $Q_{dd}$. Since it shifts all offsets equally, the lack of this term in (5-9) cannot, by itself, degrade velocity estimates. However, since the coefficient of $Q_{yy}$ depends on $d$, the time shifting done by $Q_{dd}$ indirectly changes the value of this coefficient. Fortunately, the effects of this change are small because the time shifting does not change the average value of the $Q_{yy}$ coefficient. A more important effect results from the fact that migration requires data to be downward continued until $r = \frac{\sqrt{d}}{2}$ (until the receivers are at the estimated reflector depth). The $Q_{dd}$ term changes the depth of each portion of the data such that it will be nearly correctly migrated using equation (5-8) when $r = \frac{\sqrt{d}}{2}$. Thus, if $v > \bar{v}$, the $Q_{dd}$ term shifts data to earlier arrival times. This means that the lack of $Q_{dd}$ in (5-9) will cause the data to be overmigrated (the receivers will be continued past the reflectors) when $v > \bar{v}$. This overmigration will result in an incomplete removal of velocity diffusion.
and Levin effect phenomena from the downward continued data. However, since continuation with (5-9) will partially remove these effects from the data, velocity estimates based on data continued with (5-9) should be more accurate than those based on surface data.

In summary we can conclude that the use of an incorrect downward continuation velocity will result in an incomplete removal of velocity diffusion and structurally caused moveout. For data fitting our earlier assumptions and for moderate velocity errors, the amount of structurally caused moveout remaining after migration is approximately linearly related to the degree of data over or under migration. Thus, for a continuation velocity error of 10%, approximately 10% of the surface structural moveout remains after downward continuation.

Additionally, we can conclude that, if the continuation velocity error is such that the data are not grossly over migrated (by a factor of 2 or more) velocity estimates cannot be degraded by preprocessing with downward continuation. Thus, for reasonable velocity errors downward continuation should always make some improvement in velocity estimates. In cases where the initial migration velocity is quite a bit in error, more than one iteration of migration may be necessary to obtain accurate velocity estimates.

Figures 5-8, 5-9 and parts of Figures 5-5 and 5-6 depict calculations which illustrate these results for velocity errors of 10%. The success of the migration preprocessing on the point scatterer events of Figures 5-8 and 5-9 is especially important because the linearity of the wave equation guarantees that the same quality results will be obtained for all reflector geometries.
Figure 5-8. The effect of an incorrect moveout and migration velocity.

The top two frames depict two moveout corrected common offset sections.
Figure 5-8 Cont'd.

Moveout velocity was 5500 ft/sec. The two hyperbolas were constructed with two different velocities (top is 5000 ft/sec, bottom is 5500 ft/sec). Since we are not simulating a variable velocity medium, these initial conditions should be thought of as two separate models that have been displayed on the same grid. Trace spacing is 31 ft. Time ranges from 1.52 to 2.42 seconds. The left frame is a zero offset section. The right frame is a large offset section (h=4000'). Notice that the arrival time of the apex of the deep hyperbola is independent of offset. This occurs because the true velocity of this event equals the moveout velocity. Residual moveout causes the upper hyperbola to appear late on the large offset sections.

The bottom frames show the sections after migration with the equation

\[ \frac{Q_{dr}}{d} + \frac{h}{d} Q_{yr} = -\frac{v}{8} \left(1 + \frac{4h^2}{v^2}\right) \frac{d}{d-r/v} \left(\frac{d}{d-r/v}\right)^2 Q_{yy} \]

Migration has collapsed the hyperbolas to focuses on both sections. Because the migration velocity was 10% too high for the shallow hyperbola, we should expect it to be over-migrated. The focus does have some upward curvature but the effect is small. Migration quality is not very sensitive to velocity error.
Figure 5-9. Moveout corrected common midpoint gathers for the data of Figure 5-8. The offsets (h) depicted are 0', 1000', 2000', 3000' and 4000'. The left frame depicts the surface data. Notice the residual moveout of the deeper hyperbola. Since this data was moved out with true velocity any residual moveout results from the Levin effect. The residual moveout of the upper hyperbola is larger because the moveout velocity was 10% higher than its true velocity. The Levin effect causes residual to decrease near the edge of the frame. The right frame depicts the downward continued gathers. Notice that the data corresponding to the deep hyperbola is virtually independent of offset. Levin effect residual moveout has also been removed from the shallow event. Careful measurement shows that the residual for the gather centered on the shallow scatterer is the same in both frames. True velocity can be easily estimated from the rightmost frame. Most velocity estimation procedures which are designed for surface data can be applied to the right frame without modifications.
Despite its overall success as a geophysical exploration tool, the reflection seismic method is, in some locations on earth, of little use in determining the structure of the subsurface. In these regions seismic data sections are characterized by having very little lateral coherence. Such regions are usually called no-record-areas.

It is often thought that minimal amounts of useful information about the earth can be extracted from data recorded in no record areas. In locations where the lack of data coherency is the result of poor depth penetration caused by near surface attenuation or scattering, this conclusion is probably correct. However, in areas where the poor data coherency is the result of the small coherence length of the earth itself, the use of downward continuation may make it possible to extract usable velocity information from no record data in spite of their randomness. Two geological regions which could possibly result in this latter type of no record data are heavily faulted regions and the interior regions of salt, shale or igneous intrusions.

One model of the latter type of no record area is an earth consisting of a random distribution of point scatterers. The data that would be recorded over such a model are a random function of both midpoint and time and thus appear as no-record data. Interference of events generated by scatterers located at adjacent midpoints coupled with the Levin effect may also make such data fairly complicated functions of offset. Conceivably, for some earth models the effects of interference may be large enough to cause serious degradation of velocity estimates made from surface data recorded over those earth models. In a noise free constant velocity environment this degradation might be manifested
as estimates which are either incorrect or spatially variable. Since interference reduces the total coherence of the data along the best fit hyperbola of velocity estimators, degradation might also take the form of a marked susceptibility of velocity estimates to external noise sources in a noisy environment.

By now it should be apparent that many of the difficulties associated with interference in random earth no record data can be avoided if the data are migrated before velocity estimates are made. Using this approach one can remove much of the Levin effect and interference phenomena from the data. An additional advantage of using migration is that it can accommodate the effects of reflector geometry even when the reflectors are random in three dimensions. In the three dimensional case it is necessary to regard $z$ as a radial distance from the line of the section rather than a depth.

Figures 5-10 and 5-11 show results which illustrate the properties of both surface and downward continued random point scatterer data. Figure 5-10 shows an earth model and the surface data which would be recorded over that earth model. The earth model simulates a random distribution of point scatterers which increases in density from right to left on the frame. The model was created by convolving, in time, a wavelet over a set of random numbers. The resultant data were then smoothed over midpoint with a $(1, 4, 6, 4, 1)$ filter. Smoothing was required to meet the gridding and dip restrictions of the migration equations which were subsequently applied to these data.

Frame B of Figure 5-10 shows the surface data that would be recorded over the earth model. Dip filtering and the finite scatterer size have concentrated most of the energy in Frame B in the portion of each point scatterer hyperbolic which is near the point scatterer. Thus, in spite
Figure 5-10. Random earth model and surface data. Frame A shows the earth model consisting of a 'random' distribution of point scatterers. Scatterer density increases from right to left. Frame B shows surface data generated by using Frame A as initial conditions in a time reversed version of equation (4-23). Time ranges from 2.5 to 3.5 seconds. Trace spacing is 50 feet. The moveout and migration velocity was 5000 feet/second. Dip filtering and the finite scatterer size have
limited the maximum dip in Frame B to about 20°. Such steep dips are visible on the right of the frame where the scatterer density is low. Since the earth model and the surface data appear equally random on the left of the frames, migration can be expected to produce little enhancement of midpoint coherence for these data.
of the approximately 20° dips shown on the right of the frame, most of the energy in Frame B is concentrated in low effective dip events (measured as shown in Figure 5-3).

Figure 5-11 depicts the surface and downward continued gathers constructed from the data of Figure 5-10. Residual moveout is apparent on the surface gathers especially toward the right of the frame. Residual moveout is smaller on the left where both interference and the scatterer density are large. Like residual moveout, offset coherence also decreases toward the left side of the surface data frame. Unlike the surface data, the downward continued data show no residual moveout and excellent coherence in offset.

The absence of interference and residual moveout in Frame B of Figure 5-11 indicates that velocity estimates based on the data of that frame should be superior to those based on the Frame A data. Extending these results to field data and to the earth itself, we can conclude in principle at least, that migration should allow accurate velocity estimates to be made from no record data recorded over reflectors which have little or no continuity.

Interference in Random Scatterer Data

In the previous section we discussed how migration could be used to improve velocity estimates made from random earth data and from some types of no record data. The synthetic example of Figures 5-10 and 5-11 illustrated the magnitude of improvement that could be expected for a particular earth model. Because of the small residual moveout and the high offset coherence of the surface data of Figure 5-11 an unbiased reader may question whether the improvement indicated in 5-11 is worth the cost of performing the downward continuation. To both answer this question and to gain some perspective on the effects of downward continuation in
Figure 5-11. Moveout corrected common midpoint gathers for the data of 5-10. Frame A shows the gathers constructed from the surface data. Frame B shows the gathers constructed from the downward continued data. The offsets shown are 700, 1400, 2100, 2800, 3500 and 4200 feet.
random scatterer earth models we shall need to study in detail the interference which occurs in the surface data recorded over these models.

To a large extent the importance of this interference depends on the predominant wavelength, $\lambda$, of the energy used to probe the point scatterer earth models. Even though an earth model may be random from point to point, any reconstruction of that earth model based on surface recorded reflection data must be continuous over distances comparable to the mean wavelength of the surface data. Because of this, in analyzing interference we will consider the earth to be made up of independent regions which are approximately a half wavelength square (half because we use two-way travel times). We shall call these regions pixels. (The term pixel has its origin in the field of satellite imagery where it is used to denote the smallest region resolvable on an image.)

If an earth model is not totally random we must define pixels to be larger than $\frac{\lambda}{2}$ on a side if they are to represent independent portions of that earth model. If these larger pixels are not square then directivity patterns determined by the orientation of the long axis of the pixels must be assigned to each pixel. Non-square pixels might occur if the earth were more random in the vertical direction than in the horizontal direction. Generally, because of the gridding and dip restrictions of our continuation equations, the earth models used in this thesis have pixels which are about twice as long horizontally as vertically.

Figure 5-12 is the main tool we will use in studying interference in random scatterer earth models. For discussion's sake we will be interested only in the portions of the random scatterer earth model which can contribute to the surface data recorded at midpoint $y_0$ and moveout corrected travel time $d_0$. Curve #1 of Figure 5-12 (the circle) shows the locations in the earth model which can contribute to the arrival
Figure 5-12. Plot showing the locations in an earth model which can contribute to surface data recorded at midpoint $y_0$ and moveout corrected travel time $d_0$. Curve #1 shows the locations for a zero offset section. Curve #2 shows the locations for a 45° section. In calculating curves #1 and #2 we have used $z_0 = 5000'$, and an offset of 5000' for curve #2. The pixel size shown is appropriate for a 200 foot wavelength.
at \((y_0, d_0)\) on a zero offset section recorded over the earth model. Curve \#2 (the ellipse) shows similar locations for the data arriving at \((y_0, d_0)\) on a moveout corrected large offset section \(vd_0\) \((h = \frac{v}{2} = z_0)\). Data recorded at offsets smaller than \(h = z_0\) will have contributions from points located along ellipses falling between curves \#1 and \#2.

The arrival seen at \((y_0, d_0)\) on a surface recorded zero offset section can be generated from the earth model by summing the events in the model which fall along curve \#1. Thus, much of the energy seen at \((y_0, d_0)\) on a surface recorded section, is the result of the interference of events due to scatterers located outside the pixel centered at \((y_0, z_0)\). One measure of the importance of interference in a portion of data is the ratio of the energy \(E_i\), at \((y_0, d_0)\), due to scatterers located inside the \((y_0, z_0)\) pixel, to the energy \(E_e\), at \((y_0, d_0)\), due to scatterers located exterior to this pixel. Since events due to the external pixels tend to destructively interfere we have

\[
E_e = N \cdot P
\]

where \(P\) is the mean power in a pixel and \(N\) is the number of pixels which can contribute to the data at \((y_0, d_0)\). \(N\) can be estimated by dividing the length of curve 1 by the length of a pixel.

Estimating \(N\) and using (5-11) we can get an interference measure \(I\), given by

\[
I = \frac{E_e}{E_i} = \frac{NP}{P} = \frac{2z_0 \phi}{\lambda}
\]

where \(\phi\) is a polar angle measured as shown in Figure 5-12.
Equation (5-12) is strictly valid only for zero offset data. However if offset is reasonable equation (5-12) will be fairly accurate for non-zero offset data also.

Since most reflection seismic data are such that \( z_0/\lambda \) is in the range of 10 to 100, equation (5-12) indicates that interference is quite large in point scatterer random earth data, even if the maximum values of \( \phi \) in the data are small. Figure 5-13 indicates how \( \phi_{\text{max}} \) might be defined for a particular set of data. The interference possible when \( \phi_{\text{max}} \approx 10^\circ \) is illustrated by the differences between the earth model and the surface data of Figure 5-10.

The offset to offset coherence of surface recorded random earth data may be high even though many interfering events are present in those data. The critical factor which governs coherence in offset is not how many pixels interfere but which pixels interfere to form the data seen at \( (y_0, d_0) \). If the data are such that the same pixels interfere on data of all offsets, then coherence in offset will be high. This would be the case if the data were such that interference occurred only between pixels located in regions where \( \phi \) was small (eg. the A region in Figure (5-12). Since the dip filtering and finite scatterer sizes used in generating the surface data for Figure 5-10 have limited \( \phi_{\text{max}} \) to approximately 10°, we should expect those data to have much coherence in offset in spite of the large amount of interference known to be present in them. A look at Figure 5-11 shows that the data fulfill this expectation.

Consider data of a form such that the energy arriving at \( (y_0, d_0) \) is solely due to point scatterers located in the B region of Figure 5-12. Since the length of arc on curve #1 is smaller for region B than it is for region A, data recorded in this second case will contain
Figure 5-13. If a pixel has a directivity pattern then, for it, $\phi$ has an effective maximum $\phi_{\text{max}}$. The right frame shows $\phi_{\text{max}}$ defined in terms of the data. The left shows $\phi_{\text{max}}$ defined in terms of a ray diagram.
less interference than the A region data considered earlier. In
spite of this, the B region data will be much less coherent in offset
than the A region data. This occurs because different offsets of
the B region data are the result of the interference of markedly
different sets of pixels. Correlation between adjacent offsets of
B region data can be expected only when the number of offsets recorded
is greater than the number of pixel lengths which can be measured
along a radial line intersecting curves #1 and #2 (e.g., a line like \( r, s \)
shown at the edge of the B region).

Because the various offsets of data generated by region B
scatterers can be expected to be uncorrelated, velocity estimates
based on these data will probably be random enough to be of little
value. In fact any velocity estimate can be expected to be poor as
a result of poor offset coherence if it is based solely on data due to
scatterers located in regions where the radial separation between the
ellipses corresponding to the smallest and largest offsets used in the
estimate is greater than a pixel length. In terms of Figure 5-12 we
are saying that estimates based solely on scatterers outside region A
will be poor.

In general then, we can consider any energy to be undesirable
for velocity estimation purposes if it is the result of scattering
from locations external to the region where the radial separation of
the large and small offset ellipses is smaller than a pixel dimension.
If we assume that the data generated by scatterers interior to this
region are independent of offset we can make some simple estimates
of the offset coherence of surface recorded random scatterer data. As a
coherence measure we will use an energy normalized sum over offset
called semblance. The semblance of properly moveout corrected data
along the true velocity trajectory in offset (arrival times independent of h) can be defined as

$$\text{Semblance} = \frac{(\sum_h Q(h,d_0))^2}{M \sum_h (Q(h,d_0)^2)}$$

(5-13)

where M is the number of offsets used in the sum and d_0 is the zero offset travel time at which semblance is measured.

In estimating the coherence of the random scatterer data we shall express equation (5-13) in terms of quantities which are measurable from Figure 5-12. We define these quantities as follows: S is the number of pixels having an intersection with region A; n is the total number of pixels between curves #1 and #2 which are external to region A; and J is the average number of pixels along lines like r-s in regions external to A. J enters into the calculation because some pixels external to A will be summed more than once if there are more offsets than pixels along lines like r-s. Using these definitions we can say that on average, S pixels from region A and \( \frac{n}{J} \) pixels from locations external to region A contribute to each offset of the surface data seen at \((y_0,d_0)\). Thus, we can express Q as

$$Q(h,d_0) = \frac{S}{J} \sum_{i=1}^{n/J} \xi_{ih} + \sum_{k=1}^{\mu_{kh}}$$

(5-14)

where \( \xi \) is a random variable describing the amplitude of the wave field in the A region pixels and \( \mu \) is a random variable describing amplitude in the pixels external to A. We will assume that \( \xi \) and \( \mu \) have zero mean and unit variance. Since we have assumed that data generated by region A are independent of offset, \( \xi \) is a constant function of h. Since \( \mu \) deals with pixels external to region A, it has only J degrees of freedom in offset.
To estimate semblance one need only to substitute equation (5-14) into (5-13). We will calculate the numerator first. Performing this substitution the numerator of (5-13), \( \text{NUM} \), is given by

\[
\text{NUM} = \left( \sum_{h=1}^{M} \left( \sum_{i=1}^{S} \xi_{ih} \right) + \sum_{k=1}^{n/J} \mu_{kh} \right)^2 \tag{5-15}
\]

After doing the inner sums (5-15) becomes

\[
\text{NUM} = \left( \sum_{h=1}^{M} \left( \sqrt{\frac{n}{J}} \xi_{h} + \sqrt{\frac{M}{J}} \mu_{h} \right) \right)^2 \tag{5-16}
\]

where \( \tilde{\xi} \) and \( \tilde{\mu} \) are new random variables with zero mean and unit variance. Since \( \tilde{\mu} \) has only \( J \) degrees of freedom in offset, a sum of \( M \) offsets of \( \tilde{\mu} \) can be thought of as a sum of \( J \) independent offsets each with a multiplicity of \( m = \frac{M}{J} \). Thus, after summing over offset and squaring equation (5-16) becomes

\[
\text{NUM} = \frac{m^2 J^2 S}{n/J} \xi' + 2m^2 J \sqrt{\frac{n S}{J}} \xi' \mu' + m^2 n \mu' \tag{5-17}
\]

where \( \mu' \) and \( \xi' \) are new zero mean, unit variance random variables.

Since \( \xi' \) and \( \mu' \) are uncorrelated the expected value of the numerator of (5-13), \( E(\text{NUM}) \) is

\[
E(\text{NUM}) = m^2 J^2 S + m^2 n \left( \frac{S + \frac{n}{J}}{J^2} \right) \tag{5-18}
\]

Next we will consider the denominator of (5-13), \( \text{DEN} \). Substituting equation (5-14) into equation (5-13) we have

\[
\text{DEN} = \sum_{h=1}^{M} \left( \sum_{i=1}^{S} \xi_{ih} \right) + \sum_{k=1}^{n/J} \mu_{kh} \tag{5-19}
\]

After performing the summations (5-19) becomes
\[ \text{DEN} = mJ(\ mJ S \ \xi' \mu'^2 + 2mJ \sqrt{\frac{mS}{J}} \ \xi' \mu' + mJ \frac{n}{J} \mu'^2 ) \]  

(5-20)

Thus, the expected value of the denominator of (5-13) is given by

\[ E(\text{DEN}) = m^2J^2 S + m^2 J n = M^2(S + \frac{n}{J}) \]  

(5-21)

Substituting equations (5-21) and (5-18) into (5-13) we have

\[ \text{Semblance} = \frac{S + \frac{n}{J}}{S + \frac{n}{J}} \]  

(5-22)

If we define \( \phi_{\text{max}} \) and a pixel size reasonably consistent with that \( \phi_{\text{max}} \) we can count pixels on Figure 5-12 and use (5-22) to estimate semblance. Table 5-1 shows semblance values for three values of \( \phi_{\text{max}} \).

<table>
<thead>
<tr>
<th>( \phi_{\text{max}} )</th>
<th>S</th>
<th>n</th>
<th>J</th>
<th>Semblance</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>1.</td>
</tr>
<tr>
<td>30°</td>
<td>14</td>
<td>22</td>
<td>1.5</td>
<td>.8</td>
</tr>
<tr>
<td>45°</td>
<td>14</td>
<td>60</td>
<td>3</td>
<td>.6</td>
</tr>
</tbody>
</table>

Table 5-1. Semblance values calculated from Figure 5-12 with equation (5-22).
Because we have made several assumptions in getting (5-22) the values in Table 5-1 are only approximate. Noting this we can still make statements about the value of migration in making velocity estimates from random earth data. Since correctly migrated data are independent of offset, their semblance along the true velocity trajectory is unity. Thus, on the basis of Table 5-1 we can conclude that for reasonable data parameters (offset, frequency, travel time, etc.) migration can be expected to increase the semblance along the true velocity trajectory by factors of about 50%.

On the basis of this discussion of Figure 5-12 we can make several general statements about the effects of interference on velocity estimates based on surface recorded random point scatterer earth models. First, the wave fields seen on a section recorded over a random point scatterer earth are primarily controlled by interference effects even when \( \phi_{\text{max}} \) is small. Second, interference does not greatly affect velocity estimates if \( \phi_{\text{max}} \) is small as in Figures 5-10 and 5-11. Third, velocity estimates based on surface recorded scatterer data are seriously degraded by interference when large amounts of high dip energy (\( \phi_{\text{max}} \) not small) are present. Fourth, if \( \phi_{\text{max}} \) is large, the accuracy and stability of velocity estimates based on surface recorded data can be improved if \( \phi_{\text{max}} \) is reduced by dip filtering prior to velocity estimation. Accuracy and stability can also be improved by limiting the range of offsets used in the estimates to as small an extent as possible. The amount of improvement will depend in detail on the frequency content, the range of dips and the amount of noise present in the data.
The discussion of Figure 5-12 has also shown that migration should improve velocity estimates based on random earth data, since, for typical data parameters, it increases the semblance of two-dimensional random earth data along the true velocity hyperbolic by factors of about 50%. (For earth models which are random in three dimensions the semblance increase will be greater). The significance of this increase in semblance can only be determined in the context of a particular noise model. However, the belief is that such semblance increases make velocity estimates based on migrated random earth data significantly less susceptible to the influence of noise than are estimates based on surface random earth data.

A Field Data Example

While Figures 5-5 through 5-11 indicate the usefulness of our migratory approach to velocity estimation when it is applied to synthetic data, the approach must still be tested on field data. By testing the theory of this thesis on samples of field data we hope to establish (1) that for all data examined the method does not degrade velocity estimates, (2) that the method makes needed dip and diffusion corrections to coherent but complex data and (3) that the method makes possible the measurement of velocity from laterally incoherent data. Should we fail in our third objective we have not destroyed the theoretical model. We have merely located regions on the earth where other unidentified factors, perhaps theoretical, perhaps practical, are responsible for the lateral incoherence.
As one portion of test data we shall use some 24-fold data collected near a Gulf Coast diapir. Figure 5-14 shows a stacked section of the test data. We will primarily be interested in the data arriving from the reflectors above the diapir (α in 5-14) and from reflectors interior to the diapir (Ω in 5-14). Hopefully, the discontinuous reflections above the diapir and the sparse arrivals interior to it are the result of a rather discontinuous earth structure which approximates our point scatterer model of a no record area.

Prior to making a 'before and after' comparison of velocity estimates we should briefly discuss the parameters used in downward continuing the test data. The data were migrated using equation (4-23). Thus, each offset was downward continued separately using a constant moveout and migration velocity. The velocity used was 6000 ft/sec. Velocity estimates made from the test data indicate that 'true' velocity increases from about 4800 ft/sec at 0.4 seconds to about 8000 ft/sec at 3.0 seconds. Thus, the early portions of our downward continued data will be over-migrated and the late portions of the data will be undermigrated. Finally, we note that a small amount of dip filtering was used during downward continuation to remove from the data dips which did not meet the gridding and dip restrictions of the downward continuation operator.

Figures 5-15 and 5-16 show short offset sections constructed from the surface and downward continued test data. Figures 5-17, 5-18 and 5-19 depict velocity estimates based on surface and downward continued gathers of the test data. The locations of the gathers used in making these estimates are shown in Figure 5-14. Several common midpoint gathers located at midpoints which bracket the locations of the velocity estimates are shown in Figures 5-20 and 5-21.
Figure 5-14. Field data used to investigate the advantages of migration before velocity estimation. The data shown in a 24-fold stacked section recorded near a Gulf Coast diapir (the region labeled Ω). We will be primarily interested in velocity estimates interior to the diapir and immediately above it, α. In these regions the data has poor lateral coherence and thus might be a fair approximation to no record data recorded over a random point scatterer distribution. The arrows at the top of the section show the location of the velocity estimates of Figures 5-17, 5-18 and 5-19.
surface data

Figure 5-15. A moveout corrected short offset section of the test data of Figure 5-14. The moveout correction velocity is 6000 ft/sec. Muting and AGC have been applied to these data.
Figure 5-16. The short offset section of the data of 5-15 after downward continuation. See Figure 5-21 for the significance of the arrow.
Figure 5-17. Velocity estimates based on data with high lateral coherence. The location of the gather from which these estimates were made is shown in Figure 5-14. Frame A shows the surface estimates. Frame B shows the estimates based on migrated data. Both estimates are similar for times earlier than about 1.6 seconds. After this the migrated estimates are generally lower than the surface estimates.
Figure 5-18. Velocity-time plots based on picks of the data of 5-17. The dotted curve shows estimates based on the surface gathers. The solid curve indicates the estimates based on the downward continued gathers. The estimates are essentially the same for times less than 1.6 seconds. After this, the migrated estimates are lower. A look at Figure 5-14 shows that the dip of the data increases markedly after 1.6 seconds. The velocity differences between the two estimates are in reasonable agreement with the velocity changes which can be associated with the Levin effect.
Figure 5-19. Velocity estimates in regions of poor lateral data coherence. The location of these estimates is shown in Figure 5-14. Surface estimates are shown in Frame A. Migrated estimates are shown in Frame B. Both estimates are similar for arrivals in the α region of the test data (estimates earlier than 1.8 seconds). At times later than 1.8 seconds (the Ω region of the test data) the number of 'events' and their spread in velocity appears to be smaller for the migrated estimates.
Figure 5-20. Surface and downward continued moveout corrected gathers constructed from the test data. The midpoints shown bracket the location of the velocity estimates in Figure 5-17. Only every third midpoint is shown.
Figure 5-21. Surface and downward continued moveout corrected gathers constructed from the test data. The midpoints shown bracket the location of the data used in the velocity estimates of Figure 5-19. Only every third midpoint is displayed. The arrow indicates an event which has been focused by the downward continuation. This same event is also indicated by the arrow in Figure 5-16.
There are two aspects in which the difference between surface and migrated estimates is fairly apparent. First, velocity estimates based on the downward continued data are often lower than those based on the surface data. Figure 5-18 which is based on picks made from the data of Figure 5-17 illustrates the lower estimates made from the migrated gathers. This velocity change is most apparent for estimates based on reflections with moderate to steep dip (e.g., estimates at times greater than the 1.6 sec in Figure 5-17). The magnitude of these velocity shifts is in reasonable agreement with the velocity changes which can be attributed to the Levin effect.

A second difference between the surface and migrated estimates is that there are often (but not always) fewer 'events' on the migrated estimates than on the surface estimates. Additionally, 'events' are often clustered closer together (in velocity) on the migrated estimates. These differences between the surface and downward continued estimates probably result from a combination of the effect of the dip filtering used during migration, and of the removal of velocity diffusion by the migration process.

Differences between the surface and migrated estimates are most apparent for gathers located in regions of the data where lateral coherence is high (gathers like that shown in 5-17). Where the data coherence is poor (regions labeled $\alpha$ and $\Omega$ in Figure 5-14) it is often difficult to distinguish between migrated and unmigrated estimates. Migrated and surface estimates based on data in the $\alpha$ region of 5-14 are often of equally fine quality. Like the synthetic data on the left edge of Figure 5-10, the $\alpha$ region of the test data yields good estimates
in spite of its low coherence. Theoretically, in a random earth we expect diffusion of information from midpoint to adjoining midpoint to reduce the total coherence of the surface data along the best fit hyperbolic of our velocity estimator. Hence, we expect the influence of noise to be greater on the surface estimates. This effect is not apparent, so we might conclude that the $\alpha$ region of the test data is such that the effects of interference or external noise are small. Because the $\alpha$ region of the data looks slightly layered and hence, probably has a small $\phi_{\text{max}}$, the most likely of these two conclusions is that any interference present in the data does not seriously degrade offset coherence. Both the migrated and surface estimates, based on data interior to the diapir, were fairly noisy. However, there were often fewer events and less spread of events on the migrated estimates than on the surface estimates. The lack of dramatic improvement of estimates after migration may be the result of low signal level or a poor fit of the data to our model.

On the basis of this test example, we can draw several conclusions about the use of downward continuation as a preprocessor for velocity estimation. First, when dip is small and data quality is good, downward continuation appears to have little effect on estimates and hence, does not degrade them. Secondly, when dips are moderate, and data coherence is high, downward continuation makes noticeable shifts in estimated velocities as it removes structurally caused residual moveout from the data. Finally, when the data have poor lateral coherence, migration has only a minor effect on velocity estimates if no significant interference effects are present in the data (eg., $\phi_{\text{max}}$ is small).
Chapter 6. Summary and Conclusion

Summary

A velocity estimation procedure which allows accurate velocity estimates to be made regardless of reflector geometry has been presented. The key idea of this procedure is to migrate the data before it is used to estimate velocity.

Here wave equation techniques were used to perform pre-estimation migration. In adopting a particular realization of the wave equation migration method, several simplifying assumptions were made. It was assumed that the subsurface is an acoustic medium containing only two-dimensional reflectors. This assumption allowed the use of a two-dimensional scalar wave equation to describe wave propagation. Additionally, it was assumed that the subsurface is such that multiple reflections are not significant.

Because previously published downward continuation equations were designed to model nearly vertical wave propagation, new equations valid for wide angle reflections were developed. As a first step in deriving the migration equations to be used in estimating velocity, a downward continuation equation which governs wide offset reflections generated by a single source was found. A finite difference calculation illustrated the properties of each term in this downward continuation equation. When source-receiver directivity effects were ignored this continuation equation reduced to a form similar to the previously published equations. The same calculation also indicated that wide offset reflection data can be accurately migrated with the simplified equation.
Using the techniques and insights gained from the study of the single source geometry, an equation which can be used to downward continue multi-offset sections was derived. This equation contained terms which explicitly coupled common offset sections during migration. Numerical experiments demonstrated that for many commonly encountered reflector and recording geometries, this coupling has an almost imperceptible effect on the migrated data. Additionally, synthetic data examples showed that continuation equations which neglect this coupling, model even second order effects like structurally caused moveout.

Study of some velocity estimates showed that many of the difficulties associated with estimating velocities from seismic data recorded over nonhorizontal reflectors can be traced to the fact that such data need not resemble, in detail, the earth structure over which they were recorded. On the basis of this study it was concluded that multi-offset section migration equations are just the tool needed to improve velocity estimates. Two synthetics demonstrated that, unlike estimates based on surface data, velocity estimates based on correctly migrated data exhibit no velocity diffusion effects and are independent of reflector dip and curvature. Following this, arguments stating that migration should improve velocity estimates even when an incorrect velocity is used for the migration, were presented. It was also noted that if velocity is assumed to be constant over a receiver cable length, velocity estimation programs designed for use on surface data can be applied to migrated data without modification. An additional synthetic illustrated that good velocity estimates can be made with incorrectly migrated data.
A final synthetic illustrated that downward continuation can be used to allow accurate velocity estimates to be made from no record data recorded over an earth in which the reflectors are random functions of midpoint and depth. Theoretical considerations showed that for reasonable data parameters downward continuation of random earth data could be expected to increase the semblance of that data along the true velocity hyperbolic by factors of approximately 50%.

Lastly, the velocity estimation procedure was applied to some field data recorded over a Gulf Coast diapir. These data contained regions where the reflections had good lateral coherence and regions where lateral coherence was poor. These latter regions were used to illustrate velocity estimation in no-record-areas. Estimates based on the coherent data illustrated the usefulness of migration in suppressing both diffusion effects and structurally caused residual moveout. The estimates made in the incoherent portions of the data sometimes showed minor improvements after migration. However, usually migration had no dramatic effect on the estimates. The very minor differences between the migrated and surface estimates were thought to be due to the lack of significant interference effects in that portion of the test data.

Conclusion

As stated in chapter 1, the objectives of this thesis were to demonstrate that velocity estimates could be improved by the use of downward continuation and that the problems resulting from migrating the data with an incorrect velocity could be overcome. The results of chapter 5 demonstrate that these objectives have been realized both for synthetic and field data. There we showed that continuation improves estimates because it suppresses both the Levin effect and velocity
diffusion. We also found that the fact that continuation velocities might be incorrect presents no great difficulties.

In the course of fulfilling the main objectives of this thesis we developed several additional techniques. One new capability we have gained is the ability to downward continue multi-offset sections. This ability is useful not only for our velocity estimation applications but also for improving the quality of stacked sections. The equations developed in chapter 4 can be used to migrate before stack and hence, can be used to reduce the destructive interference of signal which occurs when complex surface data are stacked.

The ability to estimate accurate velocities from some types of no record data is another capacity that we have gained along the way. If sufficient no record data fitting our three dimensionally arbitrary reflector model exists, this ability may turn out to be the fundamentally most important development of this thesis.

Although they were developed only as an introduction to the section continuation problem, the profile results developed in chapter 3 might in the long run be more useful than the multi-offset continuation equations. Recall that a 15° dip assumption was used heavily to reduce the number of terms appearing in the final section continuation equation. Study of random earth data showed that the real payoff of downward continuation occurs only when effective dips are greater than 15°. Thus, really dramatic improvements in velocity estimates resulting from downward continuation can be achieved only if the 15° assumption is relaxed by including, or at least estimating, some of these discarded terms. Unfortunately if this is done there is a distinct possibility
that the resultant finite difference algorithms may become complex and that computation costs will increase dramatically. If this is the case it may become more economical to migrate many fold multi-offset sections by using the simple profile equations of chapter 3 to migrate the data generated by each shot separately. In any event, all the equations necessary for migrating any or all of the data in a multi-offset section are contained in this thesis.
References


