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Acknowledgements

This book began as a study of a poorly understood pronominal category of Spanish—what traditional grammarians have called the pronominal neuter. Soon after the study was undertaken, it became clear that, contrary to what its name would lead one to believe, the pronominal neuter was not a third grammatical gender alongside the masculine and the feminine. If the category in question was a gender at all, it was a semantic gender: it was a category defined not in terms of form, but in terms of content. Thus began the search for a trait which would characterize the semantics of the pronominal neuter. As it turned out, the trait in question called for nothing less than a full fledged semantics of individuation. Only at this point did the book turn into what it finally came to be: a contribution to the model theory of natural language.

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1 Introduction

One of the main goals of semantics is the insightful identification of the set of meanings that human languages may express. In fact, one of the main charges of semantics is the construction of what has been aptly called the metaphysics of natural language.\(^1\) Now, as one might well imagine, the set of meanings expressed by human languages is much too vast to be identified by an exhaustive listing of all its members. Moreover, even if such a task could be completed (in an infinite amount of time) something would still have been lost in the process. The denotational domain of human languages is structured; denotations are systematically related in ways that their identification by brute listing would only conceal.

To overcome the vastness of the task and to perspicuously exhibit the structure which articulates our denotational space, the natural language metaphysician would be well advised to forego identification by listing and to adopt a recursive strategy of identification in its stead. First he should identify a set of basic objects. Then these objects should be used to construct the entire denotational domain of human language. The strategy can be illustrated by reference to the celebrated model structures advanced in Montague (1973) for a fragment of English. First, the four basic objects listed in (1) must be identified.

(1) a. The set of individuals
   b. The set of truth values
   c. The set of possible worlds
   d. The ordered set of moments of time

Then a procedure is advanced for constructing the entire denotational domain of the language (or rather the fragment thereof) under discussion. This procedure can be informally presented as follows.\(^2\)

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\(^1\) See Bach (1989).

\(^2\) Intuitively, a (mathematical) function from one set to another is a rule which assigns one and only one element from the latter to each of the elements of the
2 Linguistic Individuals

(2) a. Every individual is a possible denotation.
   b. Every truth value is a possible denotation.
   c. Every function from the set of possible denotations of a given type to the set of possible denotations of a given type is, itself, a possible denotation.
   d. Every function from the set of possible worlds at moments of time to the set of possible denotations of a given type is, itself, a possible denotation.

(2a) identifies as many possible denotations as there are individuals in (1a). (2b) identifies truth values as possible denotations. Only two such values were contemplated in Montague (1973). They are the true and the false. (2c) is one of the two properly recursive clauses in the strategy, and identifies infinitely many possible denotations. (2d) is the other recursive clause, and it, too, identifies infinitely many possible denotations.

The recursive strategy is thus capable of handling infinity comfortably. As to its ability to exhibit structure, notice that (2a) and (2b) identify what might be called the order of basic denotations. They are individuals and truth values. (2c) identifies a first kind of derived denotations. They are functions defined in terms of simpler denotations. (2d) identifies a second kind of derived denotations. They are functions defined in terms of worlds-at-moments-of-time and arbitrary denotations. It should be apparent that the recursive strategy displayed in (1) and (2) is capable of exhibiting fundamental relations between the denotations of the fragment.

But it might seem that the abstractness of the denotations in (2) renders them hopelessly underequipped to handle the denotational richness of English—let alone natural languages in general. To illustrate the descriptive power of Montague’s proposal let us see how the set of denotations defined in (2) comprises the two fundamental levels of meaning (sense and reference) of the three major kinds of expressions (terms, sentences, and predicates).³

Consider first a noun phrase like the author of Waverley. If this expression succeeds as a description in a given state of affairs, then its referent must be an individual drawn from the set of denotations identi-

³ The recognition of two levels of meaning has deep roots in the Western tradition. Modern discussion of the distinction begins with Frege (1892).
fied in (2a). But what about the sense of this noun phrase? According to Montague, the sense of this expression should be a specification of what its referent is in each state of affairs.\(^4\) For, if we knew the sense of this expression we would be in a position to say, given perfect knowledge, what its referent is in any state of affairs. But stripped to its essentials, this specification is a function from the set of possible worlds at moments of time to the set of individuals. Clearly, this function is one of the denotations identified in (2d).

Consider next a sentence like (3a).\(^5\) What does it refer to? It seems natural to suppose that the referent of an expression remains the same if we substitute one of its portions for another one which has the same referent. It follows from this assumption that (3a) and (3b) should have the same referent, since the author of Waverley and the one who wrote twenty nine Waverley novels have the same reference. But notice now that two expressions with the same sense should have the same referent. Hence, (3b) and (3c) should also have the same referent. Finally, if (3a) and (3b) have the same referent then, by the same token, (3c) and (3d) should also corefer, since there are as many Waverley novels as there are counties in Utah.

(3)  

\begin{enumerate}
    \item Sir Walter Scott is the author of Waverley.
    \item Sir Walter Scott is the one who wrote twenty nine Waverley novels.
    \item The number of Waverley novels Sir Walter Scott wrote is twentynine.
    \item The number of counties in Utah is twenty nine.
\end{enumerate}

Now, it is difficult to say what has remained constant throughout the process other than the truth value of the original sentence. If this truth value is, indeed, the only semantic aspect preserved by the process, then it follows from our first assumption that the referent of this sentence is its truth value. Truth values, of course, are the denotations identified in (2b). As to the sense of (3a), an argument entirely parallel to the one above can be constructed. Thus, if we know what a sentence like (3a) means, then we will be able to know, given perfect knowledge, whether (3a) is true or not in any given state of affairs.\(^6\) This knowledge is, essentially, knowledge of a function which assigns a truth value to each

\(^4\) This idea can be traced back to Carnap (1947).
\(^5\) This paragraph follows the discussion in Church (1956, 24f).
\(^6\) We will thus know the set of conditions under which the sentence will be true. I take it that the main tenet of truth conditional semantics is that the sense of a sentence is the set of conditions under which the sentence would be true.
possible world at every moment of time. Once again, such a function is a denotation provided for in (2d).

Consider finally an adjective like red. The referent of this adjective (in a given state of affairs) is simply taken to be the set of red individuals (in that state of affairs). But such a set may be thought of as a function from the set of individuals to the set of truth values.\(^7\) Obviously, this function is one of the denotations specified in (2c). By parity of argument, the sense of our adjective should be a specification of the individuals which are red in each state of affairs. Again, such a specification is, essentially, a function from the set of possible worlds at moments of time to the sets of red individuals. But we have just seen that sets of red individuals can be constructed as some of the functions defined in (2c). Hence the sense of our adjective is one of the denotations identified in (2d).\(^8\)

Let us assume now that the four basic objects in (1) have been identified. Let us moreover assume that the procedure in (2) has been applied so that the four objects in (1) have been used to construct an entire denotational space. How are the denotations in this space actually assigned to the expressions of a language? Once again, a recursive strategy comes to mind. First we interpret the basic expressions of the language. Then we use these interpretations to interpret complex expressions. We will thus have

\[(4) \quad \begin{align*}
\text{a. An assignment of denotations to basic expressions.} \\
\text{b. A procedure to assign denotations to complex expressions} \\
\text{given the denotations of the basic expressions they contain.}
\end{align*}\]

The four objects in (1) will be said to constitute a model when they are taken together with the assignment in (4a). The objects in (1) are said to be the structure of the model; the assignment in (4a) is said to...

\(^7\) Specifically, it may be thought of as the function which assigns truth to every individual which is red (in the given state of affairs) and falsehood to every other individual.

\(^8\) Specifically, it may be thought of as the function which assigns, to each possible world at a moment of time the function mentioned in the preceding note.

\(^9\) This strategy is neutral as to what the basic expressions of a language will be. They may well be words, but they may just as well be morphemes. In fact, even semantic features may be taken as the basic "expressions" of a language.
be the function of the model.\textsuperscript{10} To assign denotations according to the strategy just sketched is to engage in model theoretic semantics.\textsuperscript{11}

Naturally, there will be as many models as there are ways to identify the objects in (1) and to define the assignment in (4a). Some of these models may reflect reality more closely than others. Fortunately, such models need not be identified as a precondition to model theoretic semantics. Indeed, as Johnson-Laird (1983, 169) has pointed out, one of the main advantages of model theoretic semantics is that it allows us to "shelve the question of how language relates to reality and the ultimate nature of truth." Yet, it is often the case that descriptive considerations will lead a natural language semanticist to rule out a good many possible models. This is done by proposing a variety of semantic constraints. Models meeting such constraints are said to be admissible. The present study may be seen as a contribution to the identification of the set of constraints which make a model admissible.

2 The Scope of Inquiry

Let us return now to sets of individuals like (1a). Just about every formal attempt to identify the denotational space of a natural language incorporates such sets.\textsuperscript{12} Yet, until rather recently, model theoretic semantics had very little to say about them. Indeed, the only specific commitment to the set of individuals usually encountered in the literature was the requirement that the set of individuals not be empty.

The agnosticism inspired by the set of individuals can be clearly illustrated by the attitude towards 'nonactual individuals' displayed in Montague (1973, 257). Here this logician introduces a set $A$, which we may for the moment regard as the set of entities (or individuals. Or possible individuals. If there are individuals that are only possible but not actual, $A$ is to contain them; but this is an issue on which it would be unethical for me as a logician (or linguist, or grammarian, or semanticist, for that matter) to take a stand).

\textsuperscript{10} Any basic expression which has the same denotation in every model is called a logical constant. The interpretation of logical constants is usually factored out of the models.

\textsuperscript{11} Model Theoretic Semantics is a generalization of the procedure for the characterization of truth in formalized languages proposed in Tarski (1935). See Dowty, Wall, & Peters (1981) for a more detailed introduction to Model Theoretic Semantics.

\textsuperscript{12} The only exception I know of is the interesting proposal to "eliminate the universe" advanced in Keenan (1982) and work subsequent thereto. For Keenan, the structure of a model is a set of properties from which predicates draw their denotations.
While the natural language metaphysician can (and indeed should) be noncommittal about the nature of *true* individuals, he can hardly remain silent about the nature of *linguistic* individuals—the individuals which natural languages denote.

One natural language metaphysician who did not remain silent about the nature of linguistic individuals was Greg Carlson. In order to account for the semantics of the English bare plural, Carlson (1978) adopted the view that any model which is fit for doing natural language semantics must provide a set of individuals which contains a set of *kinds*. Such, he argued forcefully, were the denotations of the bare plurals we find in sentences like *Dogs bark*.

But what kind of individual is a *kind*? "If certain (unknown) conditions hold," writes Carlson (1978, 69), a series of stages are organized into an individual object. Alternatively, and possibly simultaneously, a set of stages might be organized into an individual kind. A set of objects, too, may be related to a kind in the same way a series of stages is related to an object. If certain criteria hold (which are unknown), a set of objects may be organized into a kind.

Although kinds have become part and parcel of any model worth its salt, it is clear that more needs to be known about kinds and their relation to other individuals.

Another natural language metaphysician who did not remain silent about the nature of linguistic individuals was Godehard Link. In order to account for the semantics of plurals and mass terms, Link (1983) adopted the view that any model which is fit for doing natural language semantics must provide a *structured* set of individuals. To be more specific, Link proposed that the set of individuals provided by any admissible model must have the structure of a boolean algebra which is both complete and atomic (see Appendix A).

For reasons to be presented in due course, the specifics of Link's proposal strike us as needlessly weak and complex. Yet, it is clear that the work of Link has been decisive in the sense that it has produced a consensus among formal semanticists that every admissible set of individuals must be structured. Thus, as a result of his work, the question now facing the field is not whether the set of individuals must be structured, but rather what this structure is and how should the expressions of natural languages be interpreted relative to such a domain.

The purpose of the present work is to allow the proposals of Carlson and Link to interact in the course of an argument seeking to establish that the set of individuals provided by any admissible model must be structured by the relation of instantiation (the relation of instantiation is the converse of the one which holds between kinds and their instances).
The structure which this relation imposes on the domain of discourse is that of a mereology.

Now, to say that the domain of discourse has the structure of a mereology is to say that the individuals in the domain of discourse are related as parts are related to wholes, where the parts are the instances and the wholes are the kinds (cf. Greek *meré* 'parts'). In fact, if the reader is so inclined, he may think of a kind as a (potentially discontinuous) object having its instances as (potentially disconnected) parts. Care should be taken, however, in thinking about kinds in this way. If a kind is a generic object, this object may have generic parts which are different from its instances. Thus, the generic elephant may have the generic trunk as part and the African elephant as instance. Yet, we do not wish to regard trunks and pachidermal varieties on a par. What we wish to claim is that the structure formed by parts and wholes is the same as the structure formed by instances and kinds.13

It should be noted that the relation of instantiation has been acknowledged repeatedly in the literature. It is a particular case of the relation of realization (or exemplification) proposed in Carlson (1978, 69f). It moreover coincides with the relation of representation advanced in Heyer (1985) and with the relation of realization set forth in Link (1988, 327). The proposal that the set of individuals constitutes a mereology was advanced by Massey (1976), Wald (1977) and, more recently, by Krifka (1987) and Landman (1989). The purpose of the present work is to put these two proposals together and claim that the set of individuals must constitute a mereology under the relation of instantiation. When we do so we will have contributed to the characterization of linguistic individuals. What is more, we will have provided a new basis for the semantics of individuation.

Incidentally, it might seem that the mereological proposal underlies also the study of meronomies and partonomies (cf. the structures articulating the nouns *body, arm, hand, thumb, nail*). The notion of part involved in meronomies, however, is not the one involved in our mereologies. As will become clear below, the parts and wholes of our mereologies are more 'homogeneous' than those of meronomies. More importantly, the relation relevant to meronomies may not always be transitive. Thus, an arm may have a hand and a hand may have a finger. Yet, an arm cannot be said to have a finger. Similarly, a house may have a door

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13 Clearly, a formal characterization of mereologies and a precise indication of their relevance to our claims is called for. This will be our first order of business after concluding the present introductory chapter.
and a door may have a handle. Yet, a house cannot be said to have a handle (cf. Cruse 1986, 165ff).

3 Overview of the Work

Our work will be organized as follows. In Chapter 2 we will present the reader with the fundamentals of mereological theory, formulate our claim in rather precise terms, and close with an application of the basic properties of mereologies to the construction of an elementary theory of kinds. Our presentation of mereological theory will be illustrated by a variety of nonlinguistic examples which should provide the reader with the intuitions that the formal theory tries to capture. The reader is therefore encouraged to bear in mind these examples throughout. Our claim will be formulated as part of the definition of admissible model structures. This claim will be responsible for turning a mere set of individuals into an actual universe of discourse.

Once the mereological theory of linguistic individuals has been developed, we will turn to four important applications which constitute the body of this work. We begin with the semantics of countability (Chapters 3 and 4), and continue with applications to the semantics of uncountability (Chapter 5) and, more generally, to the semantics of nominality (Chapter 6). We conclude (Chapter 7) with an elucidation of the semantics of the 'conceptional neuter', an intriguing pronominal category brought to the fore by Otto Jespersen in his Philosophy of Grammar—a remarkable source for many of the intuitions which our formal theory will try to account for.

Now, let us say that a kind is atomic (and hence improper) if it has no instances other than itself. Let us moreover say that a kind is atomistic if it is solely constituted by atomic instances. Consider by way of illustration the kind ‘elephant’. It is typically atomistic, since it is constituted entirely by individual elephants, each of which is an improper kind of elephants. Somewhere between the kind ‘elephant’ and the individual elephants, the kinds ‘African elephant’ and ‘Asian elephant’ will be found. They too will be kinds. As this manuscript goes to press, both these kinds are proper: each still has more than one instance.

Chapter 3 is based on the claim that the stem of every count noun denotes the domain of an atomistic kind, where the domain of a kind is the set of its instances. Notice that this allows the simplest of denotations for number inflections. The singular denotes the set of atomic kinds; the plural denotes the set of atomistic kinds. When a nominal stem is combined with a number inflection, a noun is formed which denotes the intersection of the sets denoted by its parts. A singular noun
thus denotes a set of atomic kinds; a plural noun denotes the domain of an atomistic kind. It should not escape the reader that this means that plurality is semantically void, or that the plural is the semantically unmarked member of the opposition of number.

Within the present setting, cardinal adjectives are interpreted along the same lines as number inflections. Each denotes the set of kinds which are constituted by a particular number of atomic instances. It follows that the cardinal adjective \textit{one} corresponds to the singular inflection; the cardinal adjective \textit{two} corresponds to the dual inflection—and so on. Similar interpretive possibilities are available to the multal adjective \textit{many} and to the paucal adjective \textit{few}.

But the semantics of countability benefits further from the mereological claim. It allows us to account for the fact, first noted in Quine (1960), that plural nouns differ from their singular counterparts in that they 'refer cumulatively'. It also allows us to provide singular and plural nouns with denotations of the same logical type (both sets of individuals). This in turn precludes the proliferation of senses for all the expressions occurring in construction with both singulars and plurals.

Also treated in this chapter are the contrasts between \textit{every} and \textit{all}, \textit{a} and \textit{some}. The first quantifiers of each series apply only to sets of atomic instances, whereas the second quantifiers of each series are not constrained in this respect. More specifically, \textit{every} will denote the restriction of the 'superset function' denoted by \textit{all} to the family of atomic sets (a superset function is one which takes a set and returns its supersets). Along the same lines, \textit{a} will denote the restriction of the 'conjoint function' denoted by \textit{some} to the family of atomic sets (a conjoint function is one which takes a set and returns its overlappers—that is to say the sets which overlap with it).

We conclude our first chapter on the semantics of countability with a discussion of the definite article. As is well known, a definite noun phrase may be taken either as a definite description or as a definite generic. Thus, an expression like \textit{the origin of the ballad} may refer either to the origin of an individual ballad we have been discussing, or else to the origin of ballads in general as a literary species. In the first case, the definite noun phrase \textit{the ballad} has been taken as a definite description; in the second, it has been taken as a definite generic.\textsuperscript{14} It will be seen in the course of this study that these two seemingly disparate uses of a definite noun phrase stem from a single interpretation of the definite ar-

\textsuperscript{14} See Jespersen (1954, Part II, §5.43).
ticle: the function which selects the mereologically greatest element from any set which has one.

As will be seen below, it follows from this and from independently motivated interpretations (i) that the denotation of a definite noun phrase is an element of the universe of discourse whenever it is defined, (ii) that the denotation of a definite noun phrase is undefined if its predicate fails to denote, (iii) that a definite noun phrase has a presupposition of uniqueness if its predicate is (semantically) singular, (iv) that the denotation of a definite noun phrase may be defined even if its predicate fails to denote uniquely, (v) that the denotation of a definite noun phrase is always defined if its predicate is plural and denotes, (vi) that a definite generic is 'defeasible' and, (vii) that there is a precise sense in which a definite generic denotes a representative of a class.

Let us suppose then that every countable stem must denote the domain of some kind, so the stem of the countable noun elephant will denote the set of instances of the kind 'elephant'. Now, as it turns out, any such domain will be a mereology in its own right. But any mereology may in fact be regarded as a set which is closed under certain well-defined operations. As such, every mereology will have subsets which are themselves closed under the operations of the mereology. Consider for instance the set constituted by the kind 'elephant', the kind 'African elephant' and the kind 'Asian elephant'. As it turns out, the set constituted by these three elements is closed under the operations which define the entire domain of the kind 'elephant'. Unsurprisingly, the subsets of a domain which exhibit such closure properties will be called subdomains.

Let us suppose now that the stem of every count noun must denote a subdomain of an atomistic kind. Notice that every domain of a kind is a subdomain thereof. But not every subdomain is a full domain (recall the set of kinds of elephant). This means that the assumption we now have in our hands is a proper generalization of the claim made in Chapter 3. As it turns out, this assumption is justified on empirical grounds. For, note that as they have been defined, domains can never be empty. As a consequence of this, the claim in Chapter 3 makes the incorrect prediction that no noun will ever denote the empty set. Since subdomains may be empty, the false prediction is avoided by the present generalization.

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15 A given set is **closed** under a given operation if the latter is a function which maps appropriate subsets of the given set **into elements of the given set**. Arithmetical addition, for instance, is closed in the set of numbers as it can be regarded as a function which assigns a number to any nonempty set of numbers.
Chapter 4 is based on a further generalization of the claim that the stem of every count noun denotes the domain of an atomistic kind. Let us for the moment focus our attention, however, on the generalization of this claim made in the preceding paragraph. Let us focus, that is, on the claim that the stem of every count noun must denote a subdomain of an atomistic kind. Notice first that the semantics of number inflection must be generalized accordingly. The singular should denote here a function which selects the set of atomic kinds of the subdomain. It will be noted that these atomic kinds need not be atomic kinds of the entire universe of discourse. Consider once again the subdomain constituted by the three kinds of elephant. The atoms of this subdomain are the kind ‘African elephant’ and the kind ‘Asian elephant’. As we write, none of these kinds is atomic in the universe of discourse.

As might be expected, the plural denotes a function which selects the set of atomistic kinds provided by a countable stem. But this is again to say that the semantic effect of the plural inflection is nil, and that the plural is the semantically unmarked member of the opposition of number. Interestingly, this means that every singular (countable) noun will denote a pairwise disjoint set of kinds—a set of kinds no two members of which share instances. A plural noun will of course denote a subdomain of an atomistic kind. These views on nouns may allow us to determine the number of (countable) nouns an admissible model may interpret (See Appendix B).

Notice that the parallelism between cardinal adjectives and number inflections is preserved within the present, generalized, setting. Each cardinal adjective will denote a function which selects the set of kinds which are constituted by a particular number of atomic instances. In fact, all the gains made for the semantics of countability of Chapter 3 will have counterparts in the semantics of countability of Chapter 4. In addition, this generalization will allow for the possibility of semantically vacuous nouns. More importantly, however, the generalization made in Chapter 4 will allow us to quantify over kinds.

Consider first a sentence like *There are but two elephants in the world*. Notice that this sentence is true if reference is made to kinds of elephants. It is false if reference is made to individual elephants. As we see it, the stem contained in the noun *elephant* is multiply ambiguous. It may denote any one of the subdomains of the kind ‘elephant’. In particular, it may denote the set of kinds of elephant with which we should be by now familiar. If the stem denotes this set, our sentence is true; if it instead denotes the entire domain of elephants, then it will be false (it will be recalled that every domain is a subdomain as well).
It should be emphasized that these 'mereological ambiguities' are strictly constrained by the algebra of subdomains; relatively few subsets of a domain may in fact qualify as a subdomain. What is more, no such senses need be listed piecemeal in the lexicon, as they will all be seen to arise by the effect of one general principle. This principle requires that every nominal stem which denotes the domain of a kind denotes also an arbitrary (but nonempty) subdomain thereof. We have called this principle The General Condition on Nominal Stems. True quantification over kinds will now be possible if quantifiers are generalized in accordance with the present setting.

But notice now that nothing said so far forces all the kinds of a universe of discourse to be atomistic. It is in fact possible for some kinds to be actually atomless, by which we mean that they have no atomic instances whatsoever. Such kinds would therefore be infinitely divisible. Chapter 5 is based on the claim that the stem of every mass noun denotes a subdomain of an atomless kind. It will be seen that this claim entails that uncountables must remain uninflected for number and that they are intolerant of cardinal modification. More fundamentally, uncountable nouns lack a principle for the individuation of their reference. In this they contrast sharply with count nouns, whose reference can be invariably 'atomized' into minimal instances.

In fact, the semantics of an uncountable noun is atomless in two distinct respects. First, it denotes the subdomain of an atomless kind. Second, the subdomain itself is atomless. These two respects are partially independent of each other. Consider for instance the kind 'red wine', the kind 'white wine', and the kind 'rosé'. Let us assume that these three kinds are pairwise disjoint, so that no portion of wine is, say, red and white at the same time. Let us furthermore assume that every portion of wine is either red, white, rosé, or a combination of any of these basic kinds. As it turns out, these three basic kinds are the atoms of a subdomain of the kind 'wine'. This subdomain will be atomistic even though it is the subdomain of an atomless kind. In a way, the entire atomless domain of the kind 'wine' is 'compressed' here into one of its atomistic subdomains.

But let us return now to the General Condition on Nominal Stems. It will be noticed that it is not restricted to the domains of atomistic kinds. It will therefore apply to atomless kinds as well. It follows that the noun wine, which is 'primarily' uncountable will have a 'secondary' countable sense—the atomistic subdomain having the three kinds of wine as atoms. We thus have an account of the well known fact that uncountables often double as countables, since the actual claim which stands at
the base of Chapter 4 is that countable stems denote atomistic subdomains. Such subdomains may contain either atomistic or atomless members.

But we also have an account of the converse fact, whereby a primarily countable noun doubles as an uncountable. Consider for instance a sentence like *He claims to be caught on the horns of a dilemma, but I see no horns nor much dilemma in his situation.*\(^{16}\) To handle such cases, a "materialization" (or "makeup") function is proposed which assigns, to each atomistic kind, its matter (or makeup). Now we propose another constraint, one which requires every noun which denotes the domain of an atomistic kind to also denote the domain of the makeup of the said kind. The second occurrence of the noun *dilemma* in our example is one such domain. It denotes the domain of the makeup of the kind whose domain was denoted by the first occurrence of this noun.

We thus provide for a veritable 'reversibility of countability'. Naturally, this is not to deny that some nouns will tend to occur as countables while others as uncountables (cf. Allan 1980). This is reflected by their inclusion in the lexicon as such. The more frequent senses will thus correspond to the 'primary' senses of the nouns.

Now, to count the number of atomic instances of an atomistic kind is a way to measure it. Such measures may then be expressed by adjectives of quantity. But uncountables can also combine with adjectives of quantity—namely with *much* and *little*. Atomless kinds should thus be measurable even though they do not have atomic instances. To account for the measurability of atomic and atomless kinds in a homogeneous way, we adapt the general measurement procedures developed for boolean algebras to the cases at hand. The result is the proposal of a 'measure function' which allows us to provide measures to atomic and atomless kinds. As a result of this, the interpretations of *much* and *little* are able to parallel closely those of *many* and *few*.

The semantics of uncountability set forth in this chapter fits in nicely with the semantics of the definite article advocated in this study. Moreover, it is able to avoid the problem posed by uncountables to the Russellian theory of definite descriptions. Simply put, the problem is this. If the world contains water, then the world will contain many portions of water. Since definite descriptions presuppose uniqueness, a description like *the water in the world* should fail, which it does not. The solution which follows from our proposals is that uncountables denote sets which are closed under 'kind formation'. They will therefore contain

\(^{16}\) See Pelletier and Schubert (1989, 398).
a single most comprehensive kind. But as interpreted in this study, it is this kind which the definite article picks.

In addition, our proposals concerning definiteness and uncountability will be able to support an account of the nonboolean conjunctions studied by Roeper (1983) and Lønning (1987). Our account will be able to avoid the bifurcation in the type system for natural language predicates adopted by Lønning (1987). In our view, natural language predicates can all denote sets. We must still bifurcate something, however: the type system of the conjunctions.

When taken in conjunction, the semantics of countability of Chapter 4 and the semantics of uncountability of Chapter 5 will be able to provide a principled account for the well known similarities between plurals and uncountables. In particular, they will be able to account for the fact that plurals and uncountables have cumulative reference, may be quasiuniversally determined by *most*, tolerate collective predication, fail to support a presupposition of uniqueness with the definite article, and contain the kinds they constitute. In each one of these respects they contrast with singulars, which fail to have cumulative reference, cannot be determined by *most*, and so on.

Now, we have seen that a kind may be atomistic. We have also seen that a kind may be atomless. It is interesting to note that atomistic and atomless kinds are mereologically homogeneous (or homomeric) in one important sense. Every instance of an atomistic instance is atomistic, and every instance of an atomless kind is atomless. Homogeneous kinds are not the only ones allowed in a universe of discourse. There may well be kinds which are mereologically heterogeneous (heteromeric) in the sense that they have both atomistic and atomless instances. Consider for instance the kind ‘red’. This kind may well have both atomistic instances (say schoolhouses) and atomless instances (say ink). We thus arrive at the following typology of kinds.
But the tolerance of mereological diversity displayed by a universe of discourse need not be matched by that of a noun. The central claim we wish to advance in Chapter 6 is that every nominal stem denotes a set of homomeric individuals.

Strong as this claim is, we believe that there are quite a number of languages which place an even stronger condition on their nouns. They are the so called 'classifier languages'. As we see them, they are characterized, at least in part, by the fact that they require its nominal stems to denote a set of atomless individuals.

Interestingly, this does not mean that classifier languages fail to provide principles for the individuation of their references. As the reader will recall, atomistic subdomains can be entirely constituted by atomless kinds. Such, we believe, is what the classifiers of these languages denote. They are nouns which denote atomistic subdomains of atomless kinds. Classifier nouns thus differ from classified nouns, which denote atomless subdomains of atomless kinds.

It follows that it should be impossible to modify a classified noun with a cardinal adjective. What is possible is to apply a classifier as a 'mediator' for such a combination. Such, of course, is the standard observation made in the description of classifier languages (cf. Greenberg 1977, 293). A classifier mediates the combination of a noun and a numeral by 'extracting' an atomistic subdomain from the denotation of the classified noun. It is to this extracted atomistic structure that the numeral applies.

We close our study with a study of conceptional neuterality, that puzzling category mentioned in Jespersen (1924, 241ff). To illustrate the category, let us consider the difference between everything and every thing. Intuitively, everything provides for a stronger type of quantification than every thing. As we see it, everything is capable of quantifying over the entire universe of discourse, whereas every thing may quantify only over the atomistic domain. Notice in particular that the red is within the domain of quantification of everything, though not in the domain of quantification of every thing. Similar points can of course be made with something and some thing, and with nothing and no thing.

Our approach in Chapter 7 will be to concentrate on Spanish, a language with a systematic expression of conceptional neuterality. This should not be taken to mean, however, that the conceptional neuter is idiosyncratic to Spanish. As far as I can see, the conceptional neuter is a hitherto neglected universal category. It is to such a category that the following comments about mass quantification should properly apply.
Mass-quantification is the most general kind of quantification. For no more general characterisation of universal quantification can be given than that it expresses that something is true of a totality, where the contrast is with something's being true of only part of the totality, or not at all. So all that this concept involves is a totality which is structured in terms of a part-of relation, and appropriate predicates. Mass-quantification has precisely this content. Thing-quantification represents a specialisation of the more general notion, arrived at by assuming an atomic structure for the totality (cf. Roeper 1983, 263).\(^\text{17}\)

The chapter concludes with the proposal of a universal set of 'mereological features' which will allow us to identify the distinctive trait of the conceptional neuter independently of language and construction.

\(^{17}\) Similar comments about uncountables may be found, for instance, in Link (1983) and Bunt (1985).
A Theory of Linguistic Individuals

1 Introduction

The term *mereology*, which means literally 'the study of parts', is due to the Polish logician Stanislaw Leśniewski, who used it to refer to the alternative to set theory he developed between the years of 1911 and 1931 in an attempt to rid mathematics from the paradox of the set of all sets which are not members of themselves (Russell).¹ In this attempt, Leśniewski was successful—at least in part.² Yet, the formulation he adopted was, as Leonard and Goodman (1940) put it, “rather inaccessible, lack[ed] many useful definitions, and [was] set forth in the language of an unfamiliar logical doctrine and in words rather than symbols.” To overcome this, Leonard and Goodman restated the theory “in a more useable form, with additional definitions, a practical notation and a transparent English terminology”; the product was then called “the calculus of individuals.” Since then, the primary interest in mereology (or the calculus of individuals) has derived from its role in the formalization of nominalist philosophy (cf. Eberle 1970), but has been applied to a variety of areas and has furthermore enjoyed a formal development of its own (cf. Simons 1987).

When used in the present context, the term 'mereology' will not be taken to refer to any philosophical doctrine which eschews sets, types, generalities, infinities, abstractions, or universals. In fact, it will not

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¹ It is believed that Leśniewski learned about this paradox in 1911 upon reading Łukasiewicz’ monograph on the principle of contradiction in Aristotle. In 1914 Leśniewski published a monograph where, according to Vito Sinisi, the mereological counterpart to sets was first adumbrated. Leśniewski published the first outline of mereology in 1916 in a monograph written in Polish and entitled “Podstawy ogólnej teorii mnogości I [= The Foundations of the General Theory of Manifolds I].” Between 1927 and 1931 Leśniewski improved on his presentation of mereology in a series of articles collectively called “O podstawach matematyki [= On the Foundations of Mathematics].” An English translation of this series will appear in Leśniewski (forthcoming), but an abridged version thereof is now available. See Leśniewski (1983).

even be used to refer, necessarily, to a theory of parts and wholes (as to sets, they have been consistently referred to in the foregoing, and will continue to be referred to in the sequel). Rather, the term 'mereology' will be used here only as the name of a particular mathematical structure—a set and a relation which exhibit certain mathematical properties. More specifically, a mereology will only be a set (call it \( E \)) and a binary relation (call it \( \leq \)) which jointly satisfy the postulates in (1).

(1) TRANSITIVITY: For all \( x, y, z \in E \): \( x \leq y \) and \( y \leq z \) jointly imply that \( x \leq z \).

COMPLETENESS: For all \( F \subseteq E \): If \( F \) is nonempty, then there exists exactly one element of \( E \) which is a sum of all elements of \( F \).

We will henceforth say that \( E \) is the field of the mereology and that its elements are the objects of the mereology. In addition, we will say that \( \leq \) is the relation of the mereology, and will let \( \leq \) define subelementhood for the mereology, so that \( x \) is a subelement of \( y \) if and only if \( x \leq y \).

Now, it will not escape the reader that the set of postulates given in (1) contains an undefined notion. It is the notion of sum involved in the postulate of completeness. To define this notion, the prerequisite notion of overlap should be addressed first. Intuitively, two objects overlap just in case there is an object which is a subelement of both. The proper formal definition of the notion of overlap is as follows.

(2) Overlap

Consider a mereology with field \( E \) and relation \( \leq \). An \( x \in E \) overlaps a \( y \in E \) if and only if there is a \( z \in E \) such that \( z \leq x \) and \( z \leq y \).

Equipped with the notion of overlap, sums can be defined as follows. Intuitively, an object is a sum of objects (henceforth 'summands') just in case two conditions are met. The first is that all summands are subelements of the sum. The second is that every subelement of a sum overlaps a summand. Formally, sums will be defined as follows.\(^3\)

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\(^3\) It is important to note that this notion of sum is stronger than that of least upper bounds in partially ordered sets. To visualize this, let us consider the diagram below.

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  w
 /|
/ \
 x y z
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Consider a mereology with field $E$ and relation $\leq$. An $x \in E$ is called a sum of all elements of some $F \subseteq E$ if and only if two conditions are met. The first is that every $y \in F$ is such that $y \leq x$. The second is that $z \in E$ and $z \leq x$ jointly imply that there exists some $y \in F$ such that $y$ overlaps $z$.

It will be noticed that the definition in (3) presupposes that $F$ is not empty, as we cannot speak coherently of 'the sum of all elements of an empty set'.

To illustrate the notion of mereology let us consider the set $\{2, 3, 5, 6, 10, 15, 30\}$ and the relation of divisibility. It is easy to show that this set and this relation jointly constitute a mereology which we may diagram as indicated in (4).

For, notice first that if some number of the set divides another, and the latter divides a third, then the first divides the third. The postulate of transitivity must therefore hold in the present context. But notice also that if $F$ is any nonempty subset of $\{2, 3, 5, 6, 10, 15, 30\}$, the elements of $F$ will have a least common multiple. But such a multiple will be unique. In addition, it will satisfy two conditions. The first is that it will be divisible by all elements of $F$. The second is that every divisor thereof will share a factor with an element of $F$. If we say that the least common multiple of all elements of $F$ is their sum, then every nonempty subset of our set will have a unique sum (in the sense of (3) above), and the postulate of completeness would also obtain in the present context.

Here $\{x, z\}$ has a least upper bound even though it has no sum. Notice that $w$ is not a sum, as $y$ is related to $w$ but overlaps neither $x$ nor $z$. The structure diagrammed above thus fails to exhibit completeness; it does not contain a sum for every one of its nonempty subsets. A partially ordered set every nonempty subset of which has a least upper bound has been called a quasi-mereology in Sharvy (1980, 621). See also Simons (1987, 88–90).
It will be noticed that each sequence $<2, 3, 5, \ldots, n>$ of prime numbers will generate a unique mereology under the relation of divisibility. For, let $\{2, 3, 5, \ldots, n\}$ be the set of primes in a sequence. Take all the nonempty subsets of this set. Find the least common multiples for the members of each of these subsets. Add these multiples to the original set. The set which results from this construction will constitute a mereology when taken in conjunction with the relation of divisibility. Clearly, each of these mereologies will have a finite number of elements. In fact, the mereology generated by a sequence $<2, 3, 5, \ldots, n>$ will have $2^k - 1$ elements, where $k$ is the length of the generating sequence.\(^4\) But prime numbers are infinite. Hence there exists an infinite family of finite mereologies.

Incidentally, notice that the empty set constitutes a mereology with respect to any binary relation. For, as defined in the preceding section, a mereology does not require that its field be nonempty. In addition, the postulates of transitivity and completeness are conditional statements. As such they are satisfied, albeit vacuously, if their antecedents are false. Thus, the postulate of transitivity will be satisfied by a relation over a set which lacks three elements. Along the same lines, the postulate of completeness will be satisfied by a set whose subsets are all empty. We will henceforth say that the mereology constituted by the empty set is the empty mereology.

Let us indulge now in a technicality and say that the empty mereology is generated by a sequence of primes of length zero. Notice that as a consequence of this technicality, the mereology constituted by the empty set can also be said to have $2^k - 1$ elements, where $k$ is the length of its generating sequence. More importantly, the technicality we have adopted will allow us to prove that every finite mereology is structurally indistinguishable from the mereology generated by some sequence of primes. These mereologies are thus illustrative of the entire family of finite mereologies.

For our next example we will leave arithmetic and turn to geometry—specifically to the set of solids in (Euclidean) space and to the ‘part of’ relation appropriate thereto. Notice that it seems natural to assume that this set and this relation jointly constitute a mereology. After all, if some solid is part of another, and the latter is part of a third, then the first should be part of the third. Hence the postulate of transitivity should hold in the present context. Moreover, if $F$ is any

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\(^4\) Caution: $k$ should not be confused with $n$. The former stands for the number of elements in the sequence while the latter represents the greatest prime in the sequence.
nonempty set of solids then it would seem that there is but one solid which satisfies two conditions: it contains all the solids in \( F \), and all its parts overlap some solid in \( F \) (a solid may have disconnected parts). If we say that this solid is the sum of all elements of \( F \), then every nonempty subset of the set of solids will have a sum (in the sense of (3) above), and the postulate of completeness would also obtain in the present context.

Let us assume then that the set of solids in space and the 'part of' relation appropriate thereto in fact constitute a mereology. Aside from its intuitive appeal, this mereology is of historical interest, as it provided the original justification for the set of mereological postulates given in (1) above. Indeed, the mereological postulates we have adopted have been drawn from Tarski (1956), where the mereology we have just suggested was used to establish the foundations of a *geometry of solids*, understanding by this term a system of geometry destitute of such geometrical figures as points, lines, and surfaces, and admitting as figures only solids—the intuitive correlates of open (or closed) regular sets of three-dimensional Euclidean geometry (cf. Tarski 1956, 24).\(^5\)

In addition to their notable compactness, the postulates proposed by Tarski have the advantage of not involving any definite descriptions. The latter is attractive since it will allow us to interpret definite descriptions mereologically in a general and insightful way. As to the compactness of the postulates advanced by Tarski, it can be gauged if we point out that these postulates entail that a mereology is indistinguishable from the positive portion of a complete boolean algebra (cf. Tarski 1956a, 333n). Moreover, a (nonempty) mereology is strictly equivalent to a complete join semilattice without a zero.

But let us return now to the mereology of solids. Consider the set of solids which are contained within any given solid. This set also constitutes a mereology under the same 'part of' relation. In fact, this mereology should be infinite, as every solid will contain further solids as parts (it will be noticed that since solids are the interiors of surfaces, a solid is never a point, a line, or a surface). But it should be clear that there will be a different mereology for each solid. Since the solids of space are infinite in number, there will exist an infinite family of infinite mereologies. As will be seen below, these mereologies are illustrative of an important class of infinite mereologies—the mereologies whose fields are the primary denotations of mass nouns.

\(^5\) The family of all nonempty regular open sets of Euclidean space constitutes a mereology when taken in conjunction with set theoretical inclusion (cf. Tarski 1956a, 341n).
2 The Mereological Theory of Individuals

It is well known that natural languages can refer to kinds as well as to their instances. Thus, the sentence in (5a) makes reference to a kind of device whereas the sentence in (5b) alludes to a particular instance thereof. Along the same lines, the sentence in (6a) makes a true statement about a species formerly found in the island of Mauritius, whereas (6b) asserts something about a specific bird.6

(5) a. Turing invented the computer.
   b. Turing repaired the computer.

(6) a. The dodo is extinct.
   b. The dodo is dead.

But if kinds and their instances are within the denotational domain of natural languages, then they must be among the denotations defined under (2) in Chapter 1. Following Carlson (1978) and others, we will henceforth assume that both kinds and their instances are in fact individuals. Kinds and their instances will therefore be among the set of denotations defined under (2a) in Chapter 1.

That instances of a kind should be individuals will come as no surprise. After all, the computer in (5b) and the dodo in (6b) are, like the author of Waverley, definite descriptions. That kinds themselves should be individuals may seem more questionable. But linguistic evidence for this claim can be found, as kinds behave like individuals in a number of characteristic respects. In particular, as noted by Carlson (1978), kinds can serve as antecedents for personal pronouns (7), they can serve as antecedents for definite descriptions (8), they can bear a name (9), and they can serve as antecedents for cataphoric pronouns (10).7

(7) [The tiger], undisturbed in the wilderness, is a majestic sight as it walks along a forest path or strides through a meadow of grass.

(8) [The muskrat], was imported into Europe in 1906. [The rodent] has since been spreading widely.

(9) The blueberry is so called because of its color.

(10) The country he loves supports [the patriot].

6 We ignore here the fact that some speakers can use dead with the same meaning as extinct.

7 A recent discussion of the biological evidence for regarding particular biological kinds (species) as individuals can be found in Mayr (1988).
But if kinds and their instances are individuals, then the set of individuals provided by a model supports a binary relation, call it the *relation of instantiation*, which relates individuals to the kinds they instantiate. The relation of instantiation thus relates each particular computer to the computer Turing is said to be the Father of; it relates the last dodo to be spotted to a species formerly found on the island of Mauritius; it relates each particular tiger to the species zoologists call *Felis tigris*, and so on.

Notice that the subkinds of a kind are kinds in their own right. Hence, the relation of instantiation should relate instances to subkinds and subkinds to kinds. Thus, in models which are consistent with Melville’s great novel, the relation of instantiation will relate Moby Dick to the sperm whale (the species *Physeter macrocephalus*), and the sperm whale to the whale (the order *Cetacea*). In fact, it will relate Moby Dick to the whale just in case it will relate it to the sperm whale.

Notice also that the relation of instantiation need not be limited to *conventionally recognized* kinds. In fact, as we see it, the relation of instantiation will relate the water in lakes, pools, drops, and molecules to the world’s water “as a total scattered object” (cf. Quine 1960, 98). Yet, drops, pools, and lakes are certainly not generally recognized kinds. In fact, nothing in the theory of kinds we are presently developing will distinguish conventionally recognized kinds from kinds *simpliciter*. Is this a shortcoming of the theory? We do not think so, since semantics should not be expected to determine what the conventionally recognized (or ‘natural’) kinds are. As we see it, that would be to expect too much of semantics and too little of particular systems of knowledge and belief.

But the case can even be made that semantics should allow for all possible kinds—the arbitrary and the perverse included—since we must be able to say that the latter are unnatural kinds. Perhaps an analogy would be useful at this point. To criticize the present theory of kinds on the grounds that it allows for unnatural kinds is like criticizing the extensional theory of properties on the grounds that it allows for unnatural properties. Although the extensional theory of properties can (and indeed should) be criticized, the grounds for criticism should not be these. As is generally admitted, every set is (the extension of) a possible property; the notion of natural class (or property) should be carved out from the notion of possible property by appropriate systems of knowledge and belief, not by the semantics of properties—after all, we must be allowed to say of unnatural classes that they are indeed unnatural.

Notice finally that we shall not assume that every kind will have instances other than itself. If need be, a distinction between kinds which
Linguistic Individuals
do (e.g., the sperm whale) and kinds which do not (e.g., Moby Dick) can
easily be made by means of the notion of mereological atom to be de-
defined below. A kind with instances other than itself may then be said to
be proper; a kind without instances other than itself may be said to be
improper (see 3.10 below). Prima facie evidence that such a distinction
is linguistically motivated is of course provided by contrasts like the
ones in (5) and (6) above.

We now wish to propose that every admissible model must provide
a set of individuals and a relation of instantiation which jointly consti-
tute a mereology. This means that to be admissible, a model must pro-
vide a set of individuals and a relation of instantiation which jointly
satisfy two straightforward conditions. One is that instantiation is
transitive; the other is that every nonempty subset of the set of
individuals constitutes a kind.\footnote{This interpretation of the postulate of completeness depends on the inter-
pretation of the notion of sum to be presented below in Section 3.5.}

More formally, our proposal is that an admissible model structure
must be defined as in (11), where $E$, $I$, $J$, $\leq$, and $<$ are respectively said
to be the set of individuals, the set of possible worlds, the set of
moments of time, the relation of instantiation, and the relation of
temporal precedence.\footnote{It will be recalled that a model structure combines with a model function to
yield a model. As might be expected, an admissible model structure combines
with an admissible model function to yield an admissible model.}

\begin{align*}
(11) & \quad \text{Admissible Model Structures} \\
& \text{An admissible model structure is a quintuple } < E, I, J, \leq, < >,
& \text{where } E, I \text{ and } J \text{ are non-empty sets, } E \text{ constitutes a mereology}
& \text{when taken together with the binary relation } \leq, \text{ and } J \text{ constit-
utes a chain when taken together with a binary relation } <.
\end{align*}

Aside from terminology, (11) departs from the celebrated model
structures of Montague (1973) in just one respect. One of the compo-
nents of the proposed model structure is a binary relation $\leq$ which con-
stitutes a mereology with $E$. We will henceforth say that a universe (of
discourse) is the set of individuals provided by some admissible
model.\footnote{We trust that the reader will not be led to believe that the relation over
moments of time is the irreflexive portion of the relation over individuals. Since
we shall have no further need to refer formally to the relation over moments, we
will not follow proper notation and endow `$\leq$' and `$<$' with different subindices.}
3 The Calculus of Kinds

We will now interpret a number of well known theorems of mereological theory (and the notions which are prerequisites thereto) in terms of an arbitrary universe of discourse and its relation of instantiation. It is hoped that the intuitiveness of these interpretations will provide preliminary motivation for our mereological theory of kinds. The list of theorems (and definitions) in question is taken essentially from Simons (1987, 37ff).

3.1 Instantiation

Let us begin with facts concerning the primitive relation of mereology.

(12.1) \( x \leq x \)

(12.2) \([x \leq y \land y \leq x] \iff x = y\)

(12.3) \( x = y \iff \forall z [z \leq x \iff z \leq y] \)

(12.4) \( x = y \iff \forall z [z \leq x \iff y \leq z] \)

(12.1), which states that the mereological relation is reflexive, means that every individual is an instance of itself. Hence, every individual is both a kind and an instance. (12.2), which states that the mereological relation is antisymmetric, means that no two (distinct) individuals can be instantiations of each other. When taken together with the postulate of transitivity, (12.1) and (12.2) entail that every mereology is a partially ordered set (= poset). (12.3) means that no two (distinct) individuals can have the very same instances. (12.4) means that no two (distinct) individuals can be instances of the very same kinds.

3.2 Overlap

It will be recalled that the relation of overlap was defined in (2), which is tantamount to (13.0). Here and henceforth, the overlap relation will be symbolized ‘\( \circ \)’.

(13.0) \( x \circ y \iff \exists z [z \leq x \land z \leq y] \)

(13.1) \( x \circ x \)

(13.2) \( x \circ y \rightarrow y \circ x \)

(13.3) \( x \leq y \rightarrow x \circ y \)

(13.4) \( x \leq y \iff \forall z [z \circ x \rightarrow z \circ y] \)

(13.5) \( x = y \iff \forall z [z \circ x \leftrightarrow z \circ y] \)

(13.0) means that two kinds overlap just in case there is an instance they share. (13.1), which states that overlap is reflexive, means that
every individual overlaps itself. (13.2), which states that the overlap relation is symmetric, means that if one individual overlaps with another then the second overlaps with the first. (13.3) means that if some individual is an instance of a kind, then the individual overlaps with the kind. (13.4) means that some individual is an instance of a kind just in case any individual which overlaps with the individual overlaps with the kind. (13.5) means that no two (distinct) individuals can overlap the very same individuals.

3.3 Disjointness

Let us turn next to the complement of the overlap relation. This is itself a binary relation. We will call it the disjoint relation, symbolize it as ‘\', and define it formally as indicated in (14.0). Some important facts about disjointness follow this definition.

\[(14.0)\]
\[JC/y<\Leftrightarrow \neg x \circ y\]
\[(14.1)\]
\[\neg x \circ x\]
\[(14.2)\]
\[x \circ y \rightarrow y \circ x\]
\[(14.3)\]
\[x \leq y \Leftrightarrow \forall z[z \circ y \rightarrow z \circ x]\]
\[(14.4)\]
\[x = y \Leftrightarrow \forall z[z \circ y \leftrightarrow z \circ x]\]

(14.0) defines disjointness as the complement of overlapping. (14.1) states that the disjoint relation is irreflexive. It means that no individual is disjoint with itself. (14.2) states that the disjoint relation is symmetric. It means that if one individual is disjoint with another then the second is disjoint with the first. (14.3) means that an instance of a kind is disjoint with every individual the kind itself is disjoint with. (14.4) means that no two (distinct) individuals can be disjoint with the very same individuals.

3.4 Proper Instantiation

Consider next the irreflexive portion of the mereological relation. Such a portion is itself a binary relation. We will henceforth use ‘<’ to refer to it in writing. Formally, < can be defined as indicated in (15.0). Some of the main properties concerning this relation follow the definition.

\[(15.0)\]
\[x < y \leftrightarrow [x \leq y \land x \neq y]\]
\[(15.1)\]
\[x < y \rightarrow \neg y < x\]
\[(15.2)\]
\[[x < y \land y < z] \rightarrow x < z\]
\[(15.3)\]
\[\neg x < x\]
Definition (15.0) means that $<$ is the relation which holds between an instance and a kind whenever the instance and the kind are distinct individuals. We will henceforth say that the former is a proper instance of the latter, and that $<$ is the relation of proper instantiation. (15.1) means that if an individual is a proper instance of a kind, then the latter is not a proper instance of the former. (15.2) means that the relation of proper instantiation is transitive. (15.3) states that the relation of proper instantiation is irreflexive. It means that no individual can be a proper instance of itself.

3.5 Constitution

It was stated in (3) that an object is a sum of objects just in case the object meets two conditions. The first is that each summand bears the mereological relation to the sum. The second is that every subelement of the sum overlaps a summand. In symbols, if $x$ is an element of a mereology, and if $\alpha$ is a nonempty subset of the mereology, then

\[(16) \quad x \subseteq \alpha \iff \forall y[y \in \alpha \to y \subseteq x] \land \forall y[y \subseteq x \to \exists z[z \in \alpha \land z \circ y]]\]

When taken in the present context, this definition means that a sum of all elements of a subset of the universe is a kind which meets two conditions. The first is that every individual in the subset must be an instance of the kind. The second is that every instance of the kind must overlap with some individual of the subset.

Intuitively, a sum of all elements of a subset of the universe seems to be a kind 'constituted' by the elements in question. We will therefore say that a kind is constituted by (all elements of) a subset of the universe whenever the kind is a sum of all elements of the subset. Before discussing the construal of 'constitution' in terms of sum, let us mention some basic facts about sums.

Consider first the postulate of completeness given in (1). It stipulates that if a mereological subset contains any elements, then these elements will have a unique sum. We may therefore define a function as follows. If $F$ is a nonempty subset of a mereology, then

\[(17.0) \quad \Sigma(F) \subseteq F\]

$\Sigma$ is therefore a function which maps each nonempty subset of a mereology into the kind constituted by all elements of the subset. One important fact about $\Sigma$ is the following. If $F$ and $G$ are nonempty subsets of a mereological field, then

\[(17.1) \quad F \subseteq G \iff \Sigma(F) \subseteq \Sigma(G)\]

(17.1) states that $\Sigma$ is an isomorphism from the set of nonempty subsets of a mereological field (as a set which is partially ordered by set
theoretic inclusion) to the mereological field proper (as a set which is partially ordered by the mereological relation). This means that mereology differs from the boolean algebra of sets merely in ways consequent upon the refusal to postulate a null element, although the [...] relation of "discreteness" [or disjointness] may be correlated with the Boolean function "x • y = 0" (cf. Leonard and Goodman 1940, 46).

In particular, our constitution function \( \Sigma \) corresponds to set theoretic union of a (nonempty) family of sets.

Let us turn now to the construal of constitution in terms of sums. The idea that a kind is a mereological sum was criticized in Carlson (1978, 102f), where mereological sums and kinds were claimed to differ in two important respects. Closer inspection of these arguments reveals, however, that the only thing these arguments sought to establish was that a kind is not the mereological sum of its parts. The arguments advanced by Carlson do not address the issue of whether a kind may be the mereological sum of its instances.* Let us inspect, then, these arguments more closely.

The first difference Carlson claimed to find between a kind and a mereological sum was that a kind may be the size of a (proper) instance, while no whole can be the size of a (proper) part. Consider for instance (18a), whose subject is taken to denote the kind of dogs. (18a) makes an assertion, albeit one which is indirect and 'generic', regarding the size of the individual dogs. Compare this to (18b), whose subject is taken to denote the mereological sum of all the asteroids in the belt. (18b) does not make any statement whatsoever about the individual size of the asteroids constituting the belt. Rather, it reports on the size of the belt proper.12

(18) a. Dogs are big.

b. The asteroid belt is big.

The second difference Carlson claimed to find between kinds and mereological sums was that kinds allow proper kind predication whereas mereological sums do not. To argue for this claim Carlson points to the naturalness of (19a), whose subject is taken to denote a kind, with the

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11 And why should they? Parts, not instances, were what mereological sums were sums of in the original formulations of Leśniewski and nominalists like Goodman.

12 The possibility that bare plurals can denote mereological sums was mentioned in passing by Burge (1972, 279). See also Cocchiarella (1976, 212), where concepts of natural kinds of 'stuff' were correlated with mereological sums.
awkwardness of (19b), whose subject is taken to denote a mereological sum of parts (i.e. territories).

(19)  a. Dogs were widespread (in 1890).

   b. ?The British Empire was widespread (in 1890).

It should be clear from this presentation that these arguments allege contrasts between kinds and sums of parts; they do not distinguish between kinds and sums of instances (the asteroids are the parts of the belt, not its instances; the territories are the parts of the empire in question, not its instances). So our claim that kinds are the mereological sums of their instances may hold in conjunction with these arguments.

Having said this, however, two observations are in order. One is that the first of these arguments establishes only that some kinds may be the size of a proper instance; it does not establish that all of them are. Indeed, as we will see in Chapter 5, some kinds are both sums of their instances and sums of their parts. These kinds cannot be the size of any proper instance. The second observation pertains to (19b). As we see it, the problem with this sentence lies not with the fact that sums of parts cannot be widespread, but rather with the fact that the British Empire refers most naturally to an individual which has no proper instances (the territories are again the parts, not the instances, of the empire). But only an individual with proper instances can be widespread. It is on these grounds that (19b) is illformed, since we will see, again in Chapter 5, that there are some kinds which can be said to be widespread in spite of the fact that they are sums of parts.

We conclude this section by providing a convenient notation for mereological sums which involve two elements of the universe. The notation is defined in (20.0). Some properties of these sums, standardly called binary, follow this definition.

\[(20.0)\quad x + y S u \{x, y\}\]
\[(20.1)\quad x + x = x\]
\[(20.2)\quad x + y = y + x\]
\[(20.3)\quad x + (y + z) = (x + y) + z\]
\[(20.4)\quad x \leq x + y\]
\[(20.5)\quad y \leq x + y\]
\[(20.6)\quad x + y \leq z \rightarrow x \leq z \land y \leq z\]
\[(20.7)\quad x \leq y \leftrightarrow x + y = y\]
3.6 The Universal Kind

Let us suppose that the field $E$ of a given mereology has some elements. The postulate of completeness ensures that all these elements will have a unique sum, one which we will henceforth refer to as $i$. For ease of reference, we will set this definition apart in (21.0).

$$(21.0) \quad i = \Sigma(E)$$

$$(21.1) \quad \exists x[x \in E] \rightarrow \forall x[x \in E \rightarrow x \leq i]$$

(21.0) means that $i$ is the kind constituted by the entire universe. (21.1) means that every member of the universe is an instance of $i$. It therefore justifies regarding $i$ as a universal kind. Incidentally, it will be recalled that the definition of admissible models advanced in (11) precludes empty universes. This ensures that every admissible model will contain one (and hence only one) universal kind.

3.7 The Void Kind

But let us suppose now that the field $E$ of a given mereology has no elements. Under such circumstances, (17.0) leaves $\Sigma(E)$ undefined. This means that the mereological theory of individuals does not have anything to say about the void kind, that hypothetical kind which lacks instances altogether. As pointed out above, this is what distinguishes a mereology from a complete boolean algebra, which not only defines the sum of an empty set, but moreover requires it to exist.

3.8 Difference

Let us suppose now that a mereological field contains two elements, call them $x$ and $y$, such that $\neg x \leq y$. It can be shown that there will exist a unique mereological difference between them. This difference will be an element of the mereological field. This element will be called $'x - y'$, and will be defined as indicated in (22.0).

$$$(22.0) \quad x - y \in \{z : z \leq x \land z \not\subset y\}$$$

$$(22.1) \quad x \circ y \rightarrow x - y < x$$

$$(22.2) \quad x \not\subset y \rightarrow x - y = x$$

(22.0) means that the difference between a minuend kind $x$ and a subtrahend kind $y$ is the sum of all individuals which are (a) instances of the minuend, and (b) disjoint with the subtrahend. (22.1) means that if a minuend and a subtrahend overlap, then their difference is a proper instance of the minuend. (22.2) means that if a minuend and a

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$^3$ Notice that $x - y$ is undefined whenever $x \leq y$. 

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subtrahend are disjoint, then their difference coincides with the minuend.

3.9 Complementation

Let us assume that a mereological field contains an element \( x \) which is not \( i \). It can be shown that the mereological field contains the mereological difference between \( i \) and \( x \). Such a difference will be denoted \( x' \) and is unique. We will call it the mereological complement of \( x \).

\[
\begin{align*}
(23.0) & \quad x' = i - x \\
(23.1) & \quad i = x + x' \\
(23.2) & \quad x \upharpoonright x' \\
(23.3) & \quad x' \subseteq \{ y : x \upharpoonright y \} \\
(23.4) & \quad x'' = x
\end{align*}
\]

(23.0) means that the complement of a kind is the difference which exists between it and the universal kind. (23.1) means that the kind constituted by any element and its complement is the universal kind. (23.2) means that any kind is disjoint with its complement. (23.3) means that the complement of a kind is the kind constituted by all the individuals which are disjoint with it. (23.4), which states that complementation is involutive, means that the complement of the complement of a kind coincides with the kind.

3.10 Atomicity

Some mereologies may have elements which bear the irreflexive of the mereological relation to no element of the mereologies. We will say that such elements are the atoms of the mereology and gather them together in a set \( A \). (24.0) formalizes this definition while (24.1) and (24.2) provide equivalent alternatives.

\[
\begin{align*}
(24.0) & \quad a \in A \iff \neg \exists x [x < a] \\
(24.1) & \quad a \in A \iff \forall x [x \leq a \rightarrow x = a] \\
(24.2) & \quad a \in A \iff \forall x [a \leq x \lor a \leq x']
\end{align*}
\]

(24.0) means here that an atom is a kind which has no proper instance. We will henceforth say that such kinds are improper; all other kinds are proper. (24.1) means that an atom is a kind which has but itself as instance. (24.2) means that every atom is an instance either of an arbitrary kind or else of its complement.

\[\text{Notice that } x' \text{ is undefined whenever } x = i.\]
4 The Concept of a Kind

We have assumed in (11) that every admissible model must provide a set of individuals. Now, by stipulation (2a) of the preceding chapter, each of these individuals is a possible denotation. But by stipulation (2d) of the preceding chapter, every function from the set of indices to the set of possible denotations of a particular type is, itself, a possible denotation. This means that every function from the set of indices to the set of individuals is a possible denotation. Since each one of these denotations has been taken to represent the concept of some individual (at least within the intellectual tradition inaugurated by Carnap, Kripke, and Montague), they have become known as individual concepts.

Now, it has been proposed that the set of individuals of an admissible model must constitute a mereology when taken in conjunction with the relation of instantiation. But this implies that the relation of instantiation is reflexive. This has been taken to mean that every individual is a kind and every kind is an individual. The set of kinds therefore coincides with the set of individuals, and the set of individual concepts coincides with the set of what may be called kind concepts. As a consequence of this, something is the concept of a kind if and only if it is the concept of some individual—at least within the distinguished tradition mentioned above.

It follows that when the mereological assumption is taken in conjunction with the independently motivated assumptions of the preceding chapter, the concepts of kinds ensue. By construing kind concepts as functions from indices to kinds we are able to distinguish between different kind concepts which happen to have the same extension in a particular state of affairs. Consider for instance any state of affairs in which sperm whales are the only extant whales. In such states of affairs, the kind of whales coincides with the kind of sperm whales. Yet, there as elsewhere, the concept of the kind of whales can be distinguished from the concept of the kind of sperm whales.

Conversely, we may be able to unify different kinds within a single concept. Consider for instance a particular world at two different times. Suppose that Moby Dick is present in one and not at the other. Although the kind ‘whale’ will change as we move from one time to the next, the

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15 An index describes the state of a world at a particular time. Formally, it is just an ordered pair constituted by two objects provided by a model: a possible world and a moment of time.
concept of the kind ‘whale’ will not: it will remain constant throughout the times (and indeed the worlds) encompassed by the model.

Now, it is plain that individual concepts can be construed as something other than functions from indices to individuals. In fact, individual concepts have been construed as objects other than these functions. The present study will not (and should not) have anything to say in this regard. What this study will do is allow us to capitalize on any theory of individual concepts (the correct one included) for it proposes that every construal of an individual concept is, ipso facto, a construal of a kind concept.\(^\text{16}\)

In his celebrated work on natural kinds, Quine (1969) wrote that “[p]roperties are intensional in that they may be counted as distinct properties even though wholly coinciding in respect of the things that have them. There is no call to reckon kinds as intensional. Kinds can be seen as sets, determined by their members.” Although we agree with Quine on the extensionality of kinds, we are forced to further recognize kind concepts—the intensional counterparts of kinds.

\(^{16}\) See Chapter 4, Section 19 for further discussion.
The Semantics of Countability I

1 Introduction

In his remarkable *Philosophy of Grammar*, Otto Jespersen pointed out that languages may contain expressions which "call up the idea of some definite thing with a certain shape or precise limits." To account for this fact, Jespersen assumed a 'world of countables', a world which is inhabited by entities like houses, horses, days, miles, sounds, words, crimes, plans, and mistakes. In the view of this chapter, the world of countables is a subset of the universe. It is the set of kinds which could be called 'atomistic' insofar as they are entirely constituted by atoms of the universe.

To gauge the descriptive power of this view, let us recall that an atom of a mereology is, by definition, an element which has no proper subelements. Let us moreover say that a mereology is atomistic just in case every one of its elements has an atomic subelement. More formally, let $E$ be a set and $\leq$ be a binary relation which jointly constitute a mereology. This mereology is *atomistic* if and only if for every $x \in E$ there is an atom $a$ of the mereology such that $a \leq x$. A universe is therefore atomistic if and only if every kind of the universe has an atom as instance.

Consider for example the mereology of primes generated by the sequence $<2, 3, 5>$ of primes mentioned in the preceding chapter. This mereology has three atoms—the members of the sequence. Moreover, the mereology is atomistic, since every one of its members is divisible by one of the primes in the sequence. In fact, all the mereologies of primes discussed in Chapter 2, even the empty mereology of primes, are atomistic. Even more generally, *all finite mereologies are atomistic*.

The atomisticity of the mereologies of primes should be contrasted with the nonatomisticity of the mereologies of solids also mentioned in the preceding chapter. For, it will be noticed that these mereologies contain no atoms, as there are no least inclusive solids in space.

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1 See Jespersen (1924, 198).
Consequently, none of the mereologies of solids introduced in the preceding chapter can even begin to be atomistic.

To illustrate the concepts and interpretations to be advanced in this chapter we will now introduce an intuitive (albeit abstract) example. Consider a universe whose mereological structure can be diagramed as indicated in (1). As a glance at the diagram under (4) in Chapter 2 will reveal, this mereology is isomorphic to the mereology generated by the sequence \(<2, 3, 5>\) of primes, and is obviously atomistic.

(1)

\[
\begin{array}{c}
a+b+c \\
a+b \quad a+c \quad b+c \\
a \quad b \quad c
\end{array}
\]

2 Countable Stems

To provide an interpretation for the stems of countable nouns we will invoke the domains (or principal ideals) in a universe. Intuitively, a domain is the set constituted by all the instances of a kind. More formally, let \(E\) be a universe and let \(k\) be an element of \(E\). The domain of \(k\) in \(E\) is the set \(E \mid k\), which we define in (2).

(2) \(E \mid k = \{x \in E : x \leq k\}\)

It will be recalled that we have assumed under (11) in Chapter 2 that universes are not empty. Hence every universe will contain at least one domain—the domain of the universal kind.

Take for instance the universe in (1). There will in fact be seven domains in this universe. They are the following.

(3) \(E \mid a = \{a\}\) \(E \mid a+b = \{a+b, a, b\}\) \(E \mid a+b+c = E\)
\(E \mid b = \{b\}\) \(E \mid a+c = \{a+c, a, c\}\)
\(E \mid c = \{c\}\) \(E \mid b+c = \{b+c, b, c\}\)

Notice that there will always be as many domains as elements in a universe. Notice also that every universe is the domain of its universal kind. Notice finally that no domain can be empty, and that \(E \mid k\) is a singleton if and only if \(k\) is an atom (hence an improper kind) of \(E\).
Now, it should be clear that every domain will be a mereology in its own right. As such, it may be atomistic. We may therefore speak coherently of the atomistic domains, if any, in a universe. Consider for instance the seven domains in (3). Since they are finite, they are atomistic. We may therefore speak of the seven atomistic domains in (1).

The semantic relevance of atomistic domains lies in (4), a semantic condition which constitutes a further constraint on the set of models which are fit for describing the semantics of natural languages. The condition in (4) should thus be part of any adequate definition of admissible model.\(^2\)

\((4)\) **Countable Stems**

Every countable stem denotes an atomistic domain in the universe.

To illustrate the strength of (4), let us observe that the universe depicted in (1) has one hundred twenty eight subsets. Of these, only the seven sets listed in (3) may serve as countable stem denotations. More generally, any universe with \(k\) atomistic kinds will have \(2^k\) atomistic subsets but only \(k\) possible countable stem denotations.

We close this section with an observation pertaining to 'denotation failure' in atomistic universes. Recall first that no domain can ever be empty. It follows from this and (4) that no countable stem denotes the empty set in an atomistic universe. This proposition therefore entails that no countable stem can ever 'fail to denote' in an atomistic universe. Consider for instance the stems in the nouns *unicorn*, *centaur*, or *mermaid*. As (4) would have it, all these stems would have to denote individuals in any atomistic universe. Thus, whenever these or any other countable stems 'denote', they would do so out of logical necessity!

To escape this difficulty, one could of course impose *ad hoc* riders on condition (4). One could, for instance, restrict (4) to countable stems which denote a nonempty set. Instead of doing this, however, we will postpone further discussion of the issue until next chapter, where (4) will be generalized in a natural and independently motivated way. The intended generalization will be such that countable stems will be able to denote the empty set.

\(^2\) It will be recalled that only the characteristic function of a set of individuals, not the set itself, is an admissible denotation. But sets and their characteristic functions are interdefinable. To enhance readability, our interpretations will henceforth appeal only to sets of individuals. The interested reader may always translate such sets into the language of functions. See Chapter 4 for the final versions of all the semantic conditions presented in this chapter.
3 Number Inflection

The condition on the interpretation of stems provided in the preceding section allows us to propose the simplest of interpretations for number inflection. To do so, however, we should first point out that we shall take the term ‘inflection’ in its broadest possible sense. In particular, the term will be intended to encompass the juxtaposition of an overt affix to a stem (cf. the suffix on rings), the replacement of a portion of a stem (cf. the vowel of men), both affixation and replacement (cf. the changes in stem vowel and ending on children), and neither affixation nor replacement (cf. the phonetically null inflection on the plural sheep).

Let us propose then (5), which provides universal interpretations for number inflections. It should not escape the reader that to provide universal interpretations for number inflection is to claim that number inflections are logical constants—expressions whose interpretations do not vary from model to model, but are rather fixed once and for all in universal grammar. As might be expected in (5), an element of the universe is atomic if it is an atom of the universe; it is atomistic if it is entirely constituted by atoms of the universe.

(5) Number Inflection
   a. The singular inflection denotes the set of atomic elements of
      the universe.
   b. The plural inflection denotes the set of atomistic elements of
      the universe.

To illustrate, let us consider a model which provides the universe in (1) and abides by (5). This model would interpret the singular inflection as the set whose members have been enclosed in Figure 1, while assigning the set whose elements have been enclosed in Figure 2 to the plural inflection.
It will be noticed that the plural inflection has been made to contain all the atomistic elements of the universe, the atoms included. Evidence for this decision will be provided in Section 6 below.

The effect of the interpretations in (5) will be best appreciated in the next section, where the interaction of stems and inflections will be discussed. Suffice it to say at this point that the interpretations in (5) can be easily extended to account for the dual inflections of languages like Classical Greek, Greenlandic, and Sanskrit, for the trial inflections of languages like Fijian, and for the quadral inflections of languages like Sursurunga. Consider for instance (6), where the terms monatomic, diatomic, triatomic, tetratomic are used in strict accordance with their etymologies.

(6) **Number Inflection (Generalized)**

a. The singular inflection denotes the set of monatomic elements of the universe.

b. The dual inflection denotes the set of diatomic elements of the universe.

c. The trial inflection denotes the set of triatomic the elements of the universe.

d. The quadral inflection denotes the set of tetratomic elements of the universe.

e. The plural inflection denotes the set of atomistic elements of the universe.

It should be clear that any model which provides the universe in (1) and abides by (6) would assign the set in Figure 3 to the dual inflection, the set in Figure 4 to the trial inflection, and the empty set to the quadral inflection (there are no tetratomic elements in this universe).

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*Sursurunga is a Patpatar-Tolai language spoken in Papua New Guinea. See Hutchisson (1986) for a discussion of its quadral number.*
We conclude this section by introducing a notational convention which will contribute significantly to the perspicuity of our presentations. As any notational convention, it plays a role only in the presentation of a theory, not in the theory itself.

(7) Asterisk Convention:
Let $P^*$ be the set of elements which are constituted by elements from some subset $P$ of the universe.

It will be noticed that $P^*$ is an atomistic domain whenever $P$ is a set of atoms, and that $P^*$ will be empty whenever $P$ is. It will also be noticed that $A^*$ is the set of atomistic elements of the universe whenever $A$ is the set of atomic elements of the universe.  

4 Countable Nouns

A countable noun is the combination of a countable stem and a number inflection. Since countable nominal stems and number inflections both denote subsets of the universe, the interpretation of a countable noun will be governed by the following universal semantic principle.

(8) The Intersection Principle
Whenever two expressions which denote subsets of the universe combine, the denotation of the resulting expression is the intersection of the two subsets.

As might be expected, the Intersection Principle is motivated independently of the matters at hand, as it governs the interpretation of a variety of expressions whose parts can combine meaningfully in just one way. \footnote{The Intersection Principle thus amounts to an instance of what Klein & Sag (1985) have called type driven translation. This principle will be invoked repeatedly in the sequel.}

Be that as it may, it should be noticed that we regard this principle as a markedness convention which will be automatically overruled by any specific stipulations which are incompatible with it.

To illustrate, let us consider (9), which contains the interpretations of two countable nouns relative to a model which abides by conditions (4), (5), (8). Here we assume that these nouns involve a countable stem \textit{ring} and that $R$ is the set which collects those atoms of the model which are in fact rings. \footnote{Following standard usage, we will henceforth use $[[a]]$ for the denotation of an expression $a$ (relative to a model whose identity is obvious from the context).}

As convened, $A$ is the set of atomic elements of the universe.

Footnotes:

4 Technically, $P^*$ is $\{Q(Q): Q \subseteq P \text{ and } |Q| \geq 1\}$, the closure of $P$ under mereological sum.

5 Following standard usage, we will henceforth use $[[a]]$ for the denotation of an expression $a$ (relative to a model whose identity is obvious from the context).
Let us say now that the interpretations in (9) have proceeded relative to a model which provides a universe which can be diagramed as indicated in (1) above. In addition, let us say that this universe contains only two rings, namely a and b. Relative to this model, the singular noun ring will denote the set represented in Figure 5 below. The noun rings, on the other hand, would denote the set represented in Figure 6.

To further illustrate the interaction of countable stems and number inflections, let us turn to (10), the interpretations of two countable nouns relative to a model which satisfies conditions (4), (5), (8). Here we assume that these nouns involve a countable stem thing which simply denotes A*, the set of atomistic elements provided by the model. It will be noticed that the noun thing denotes A, the set of atomic elements of the model in question.7

Relative to our paradigmatic universe, the singular noun thing will therefore denote the set in Figure 1 above, whereas the plural noun things will refer to the set in Figure 2.

Our final illustration will come from Classical Greek. Consider first (11), which contains the interpretations of the singular daktulios 'ring', the dual daktyliō 'two rings', and the plural daktulioi 'rings'. The interpretation proceeds relative to a model which satisfies conditions

Also following standard practice, we assume that the number inflection of singular nouns is phonetically null in English.

7 Hence the name 'thing-words' which Jespersen (1924) uses for countables.
(4), (6), (8), provides a set \( R \) of rings, and provides a set \( 2R \) of binary sums of rings.

\[(11) \]
\[\text{a. } \llbracket \text{daktulios} \rrbracket = \llbracket \text{daktuli} \rrbracket \cap \llbracket \text{os} \rrbracket = R\]
\[\text{b. } \llbracket \text{daktyliō} \rrbracket = \llbracket \text{daktyli} \rrbracket \cap \llbracket \text{o} \rrbracket = 2R\]
\[\text{c. } \llbracket \text{daktulioi} \rrbracket = \llbracket \text{daktuli} \rrbracket \cap \llbracket \text{oi} \rrbracket = R^*\]

If the universe against which the nouns in (11) have been interpreted can be diagramed as indicated in (1), and if the universe contains only two rings \( a, b \), then the singular noun \( \text{daktulios} \) will denote the set whose elements have been enclosed in Figure 5 above; the dual noun \( \text{daktyliō} \) will denote the set whose elements have been enclosed in Figure 7 below, and the plural noun \( \text{daktulioi} \) will denote the set enclosed in Figure 6 above.

![Figure 7](image)

### 5 Singularity as Atomicity

It should be clear that conditions (4), (5), (8) jointly imply that every singular noun will denote a (nonempty) set of atoms of the universe. Conversely, every (nonempty) set of atoms is a potential singular noun denotation. It follows that singularity is atomicity or, more precisely, that a noun is singular if and only if it denotes a nonempty set of atoms of the universe. Consequently, any model which abides by conditions (4), (5), (8), and provides an atomistic universe with \( k \) atoms will provide \( 2^k - 1 \) possible singular noun denotations (one for each nonempty set of atoms).

It should also be clear that conditions (4), (5), (8) jointly imply that every plural noun will denote the closure of a (nonempty) set of atoms under mereological sum. Conversely, every closure of a nonempty set of atoms under mereological sum will be a potential plural noun denotation. It follows that plurality is atomic closure or, more precisely,
that a noun is plural if and only if it denotes the closure of a nonempty set of atoms of the universe under mereological sum. Since each of these closures is distinct, any model which abides by conditions (4), (5), (8), and provides an atomistic universe with $k$ atoms will again provide $2^k - 1$ possible plural noun denotations (one for each nonempty set of atoms).

Now, as it turns out, every atomistic universe with $k$ atoms will itself have $2^k - 1$ elements. Hence any model which abides by (4), (5), (8) and provides an atomistic universe with $n$ elements will provide $n$ possible singular noun denotations and $n$ possible plural noun denotations.

6 Markedness and Plurality

Let us say now for the sake of brevity that \[
[SINGULAR] \quad \text{is the denotation of the singular inflection while } [PLURAL] \quad \text{is the denotation of the plural inflection. We observe that the inclusion in (12) will hold in any model which abides by (5).}
\]

(12) \[ [SINGULAR] \subseteq [PLURAL] \]

But the converse of (12) will not hold in these models. We conclude that the plural is the semantically unmarked member of the opposition of number in every model which abides by (5).

The idea that plurals may be semantically indeterminate with respect to number is not new. In fact, it was observed in McCawley (1968, 568) and Krifka (1987, 10) that the plural is normal when it is not known whether one or more than one individual is being referred to. Thus, notice that application forms give plural headings like *schools attended*, *previous positions*, and *children*—their singular counterparts clearly presupposing only one of each.\(^8\)

Furthermore, notice that the nouns of nominal comparatives are plural and denote without prejudice against singularity. Thus, it is possible for the sentence in (13a) to be true even when there is just one question and just one answer. Similarly, it is possible for the sentences of (13b) to be true even when there is just one question or just one answer (though not both, of course).

(13) a. We always have as many questions as answers.

b. We always have more/less questions than answers.

Finally, there are the plural of royalty (cf. *We Victoria Regina*), the plural of modesty (cf. *we rest our case*), the colloquial *we* of solidarity

\(^8\) The idea that plurals are the unmarked member in the opposition of number was also defended in Roberts (1986, 174ff).
(cf. *we'll see you tomorrow*), and the plural of politeness (cf. the use of *y'all* for a single addressee in Southern English or the use, in Hindi, of plural pronouns and verb forms in all grammatical persons as an expression of politeness to the addressee). In all these cases, a plural pronoun may be used to refer to a single individual.

But having argued that the true meaning of plurality is as indicated in (5b), we must be prepared to acknowledge the effects of a rather general process whereby the unmarked term of an opposition can come to denote *the semantic difference* between the marked and the unmarked terms. Cast in the original Praguean terms, this is the process whereby an expression which has a general meaning (*Gesamtsbedeutung*) develops a specific meaning (*Grundbedeutung*). Thus the plural inflection, whose general meaning is the set of atomistic elements, may come to develop a specific meaning, namely the set of polyatomic atomistic elements. Consider first a model in which the general meaning (*Gesamtsbedeutung*) of the plural noun *rings* is the set enclosed in Figure 6. The point just made entails that we are prepared to acknowledge the existence of a specific meaning (*Grundbedeutung*) for this noun. Relative to this model, this is the set enclosed in Figure 7. Or consider a model in which the general meaning of the plural noun *things* is the set enclosed in Figure 2. We should not be surprised to find that such a noun will develop, relative to the same model, a specific meaning amounting to the set enclosed in Figure 8 below.

Figure 8

It should be clear that this is not the place to determine the conditions under which a specific meaning will develop. Suffice it to say at this point that an expression will have a general meaning unless the specific meaning is ‘forced’ by (linguistic or extralinguistic) context. The specific meaning of an expression will hence arise when this expression combines with another which ‘selectively restricts’ for the specific

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9 See Waugh (1976, 94–98) and the references cited therein.
meaning. Actual examples of this phenomenon will be provided below—for instance when we discuss (14).

(14) All rings look alike.

Since only polyatomic individuals may look alike, the plural noun *rings* must develop its specific meaning if (14) is to be true.

Let us return now to the main point of this section. It might seem that the unmarkedness of plurality stated in (12) and entailed by our proposals is contradicted by the fact that only plurals, not singulars, tend to be overtly marked in the languages of the world (cf. Greenberg 1966, 28). But this appearance of contradiction will vanish if we distinguish between syntactic and semantic number. Notice first that there is ample evidence for the distinction: nouns like *scissors*, *trousers*, *tongs*, and *goggles* may be syntactically plural even when they are semantically singular. Conversely, noun phrases like *many a man* and *more than one writer* may be syntactically singular even though they are semantically plural. But equipped with the distinction between syntactic number and semantic number we may claim, following McCawley (1977, 376), that even though syntactic plurality may be more marked than syntactic singularity, semantic singularity will be more marked than semantic plurality. Hence the widespread overt marking of the plural may only contradict a claim to the effect that plurals are syntactically less marked than singulars. This marking does not contradict the claim that plurals are semantically less marked than singulars.

Still, the fact that only the semantically unmarked plural tends to be overtly marked calls for an explanation. After all, according to Jakobson (1963, 586), languages tend to avoid any mismatch between pairs of unmarked/marked categories, on the one hand, and pairs of zero/nonzero allomorphs on the other. One such explanation is again due to McCawley, and runs as follows:

Reasons of efficiency dictate that if one number is to be morphologically unmarked, it will be the singular: the occasions on which one is talking about one object outnumber occasions on which one is either talking about more than one or talking in general terms that cover both one and more than one, and thus, fewer morpheme tokens will be used if singular is morphologically unmarked and plural marked than vice versa (cf. McCawley 1979, 270).

Thus, the commonness of the meaning of singularity would overrule the semantic markedness of this meaning and would account for the syntactic unmarkedness of singularity.

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Naturally, the frequency claim made by McCawley should be supported by statistical evidence before it can be granted. Moreover, if true, the claim would mean that the singular would be both more marked (semantically) and more frequent than the plural. Yet, it has been argued that markedness is inversely proportional to frequency: the more marked a feature is, the less frequent it is (cf. Greenberg 1966). Clearly, the theory of plurality presented in this chapter cannot succeed unless we meet these challenges. It is hoped that the remainder of this study will provide the motivation to do so.

7 Cumulative Reference

Let us turn then to cumulative reference, a property which Quine characterized as he attributed it to mass terms:

So-called mass terms like ‘water’, ‘footwear’, and ‘red’ have the semantical property of referring cumulatively: any sum of parts which are water is water (cf. Quine 1960, §19).

But mass nouns are not the only nouns which may refer cumulatively. As has been noted, plural nouns also have cumulative reference, as sentences like (15a) and (15b) imply sentences like (15c). Naturally, we assume here that the denotations of the conjoined subjects of (15a) and (15b) are both instances of the denotation of the conjoined subject of (15c).

(15)  a. Charles I and Charles II are kings.
     b. Charles III and Charles IV are kings.
     c. Charles I and Charles II and Charles III and Charles IV are kings.

To be precise about the property of cumulative reference, let us observe that every universe is by definition complete (it satisfies the mereological postulate of completeness, whereby every universe must contain the mereological sum of all elements of each one of its nonempty subsets). But this notion of completeness may be extended to arbitrary subsets of the universe in an obvious way: a subset of the universe may be said to be complete just in case it contains the mereological sum of each one of its own nonempty subsets. We may now define formally ‘the semantical property of referring cumulatively’ as follows.

(16) Cumulative Reference

An expression will be said to have cumulative reference just in case it denotes a complete subset of the universe.
But (4), (5), (8) imply that every plural noun has cumulative reference (atomistic domains are mereologies in their own right; they are therefore complete). The facts in (15) can thus be accounted for.

Incidentally, it should be noticed that the property of cumulative reference is not trivial, as not all nouns will satisfy it. Consider for instance singular nouns. As we have seen, every singular noun will denote a set of atoms. But a set of atoms will denote cumulatively just in the trivial case that such a set has exactly one member. Thus (17c), whose subject denotes the sum of the denotations of the subjects of (17a) and (17b), is illformed as expected, even if (17a) and (17b) are true.

(17) a. Charles I is king.
   b. Charles II is king.
   c. ?Charles I and Charles II is king.

That the illformedness of (17c) is semantic rather than syntactic follows from the wellformedness of (18), where a conjoined subject tolerates a predicate nominal in the singular.\footnote{James McCawley has called to my attention, however, the acceptability of \textit{Christian XI and Zog IV have been king for only a few years}. The cumulative reference of mass nouns will be taken up below.}

(18) Charles I and only Charles I is king.

8 Semantic Parsimony

The theory of plurality presented in this study is based on the claim that the set of individuals provided by a model is structured. As has been pointed out, this claim was advanced and defended in Link (1983). Before the work of Link, the formal theory of plurality in current use was the one advanced by Bennett (1974, 88ff). Here a plural common noun was assigned the set of nonempty subsets denoted by its singular counterpart. Thus, relative to a universe with the mereological structure in (1) and with two atomic rings $a, b$:

\begin{align*}
(19) & \text{ a. } & [\text{ring}] = \{a, b\} \\
 & \text{ b. } & [\text{rings}] = \{\{a\}, \{b\}, \{a, b\}\}
\end{align*}

But this means that singular and plural nouns belong to different logical types; singulars denote sets of individuals while plurals denote sets of sets of individuals. Now, there is a wide range of expressions in English which combine with nouns regardless of their semantic number (cf. for instance the definite article \textit{the}, the nonexistential quantifier \textit{no}, and ordinary adjectives like \textit{expensive}). It follows that these expressions would have to be in effect ambiguous so as to combine with singulars under one sense and with plurals under another. Thus, two adjectives
expensive would have to be distinguished: expensive₁ would be a set of individuals and can therefore intersect singular nouns (but not plural nouns); expensive₂, on the other hand would be a set of sets of individuals and could therefore intersect plural nouns (but not singular nouns). Needless to say, there is no independent semantic motivation for such a proliferation of senses; there are no distinctly singular or distinctly plural ways of being expensive.¹²

The only way to attain a unified analysis of nouns under this theory is to generalize to the worst case and assign sets of sets also to singulars. Thus, relative to the said model, ring and rings would denote as shown in (20). But then no independent motivation can be found for the increment in abstractness indulged in by (20a).

(20) a. \([\text{ring}] = \{\{a\}, \{b\}\}\)  
b. \([\text{rings}] = \{\{a\}, \{b\}, \{a, b\}\}\)

9 Cardinal Adjectives

Notice now that every atomistic kind of the universe is a sum of exactly one set of atoms. It follows that we can define a function which assigns, to each atomistic kind, the number of elements (or cardinality) of this set. Suppose we were to in fact define one such function and call it ‘\(\mu\)’. Our function \(\mu\) will count the number of atoms which constitute any atomistic kind, as it maps atomic elements to the number one, diatoms to the number two, triatoms to the number three—and so on.²⁻¹³

Equipped with \(\mu\), we may now propose universal interpretations for cardinal adjectives. Notice that we wish to remain neutral with respect to the manner in which the denotations of these adjectives are generated by the various grammars. Clearly, most of them should be generated compositionally from the meanings of their parts, and this can be done straightforwardly. As usual, \(E\) stands here for the universe.

¹² There certainly are languages whose adjectives and determiners exhibit number variation. Following standard practice, we regard this variation as an effect of concordance—a strictly syntactic matter which is of no semantic consequence.

¹³ It should be clear that \(\mu\) will tell us the ‘height’ at which each element of an atomistic universe will be drawn in a mereological diagram—atoms at the lowest, diatoms at the second lowest, and so on. \(\mu\) is therefore normally referred to as the ‘height function’ \(h\). Unfortunately, the height function does not apply to nonatomistic elements of the universe. Since we indeed wish to encompass such elements and have the same function ‘measure’ both (see Chapter 5), we have adopted in the text any function \(\mu\) which will in effect coincide with \(h\) for atomistic elements, but is not by definition limited to them.
(21) **Cardinal Adjectives**

The cardinal adjective $n$ denotes the set $\{x \in E: \mu(x) = n\}$, where $\nu$ is the $n$th positive integer.

Thus, every cardinal adjective $n$ denotes the set of kinds constituted by $n$ atoms. To illustrate, notice that any model abiding by conditions (4), (5), (8), (21) will induce the interpretations in (22). As might be expected, $A, (2A, 3A, \text{etc.})$ stand for the set of atoms (diatoms, triatoms, etc.) of the universe. Furthermore, $R (2R, 3R, \text{etc.})$ is the set of atomic (diatomic, triatomic, etc.) rings in $A (2A, 3A, \text{etc.})$.

(22) a. \([\text{one ring}] = \{\text{one}\} \cap [\text{ring}] = A \cap R = R\)
   b. \([\text{two rings}] = \{\text{two}\} \cap [\text{rings}] = 2A \cap R = 2R\)
   c. \([\text{three rings}] = \{\text{three}\} \cap [\text{rings}] = 3A \cap R = 3R\)

Relative to our paradigmatic universe, (22a) would be the set represented in Figure 5. (22b) would be the set represented in Figure 7 instead. Interestingly, (22c) will be the empty set, as there are no triatomic rings in this universe. In addition, the model in question will provide for the following interpretations.

(23) a. \([\text{one thing}] = \{\text{one}\} \cap [\text{thing}] = A \cap A = A\)
   b. \([\text{two things}] = \{\text{two}\} \cap [\text{things}] = 2A \cap A = 2A\)
   c. \([\text{three things}] = \{\text{three}\} \cap [\text{things}] = 3A \cap A = 3A\)
   d. \([\text{four things}] = \{\text{four}\} \cap [\text{things}] = 4A \cap A = 4A\)

Relative to the usual universe, (23a) would be the set represented in Figure 1 above, while (23b) and (23c) would be the sets represented in Figures 3 and 4 respectively. (23d) is the empty set, as the universe contains no four things.

Finally, notice that, as might be expected, \([\text{one}], [\text{two}], [\text{three}], [\text{four}],\) are respectively equal to \([\text{SINGULAR}], [\text{DUAL}], [\text{TRIAL}], [\text{QUADRAL}],\) the denotations of the number inflections given in (6) above; numerically modified nouns and number inflected nouns thus have denotations of the same type—subsets of the universe of discourse (cf. Link 1987).

10 Multal and Paucal Adjectives

But our function $\mu$ also plays a role in the interpretation of the paucal adjective few and the multal adjective many.

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14 It should be noted that this is so even though rings denotes a set with three elements.
Linguistic Individuals

(24) Multal and Paucal Adjectives (Countable Version)

a. The adjective many denotes the function which assigns, to each atomistic domain \( P \), the set \( \{ x \in P : \mu(x) \text{ is large in } P \} \).

b. The adjective few denotes the function which assigns, to each atomistic domain \( P \), the set \( \{ x \in P : \mu(x) \text{ is small in } P \} \).

Thus, whenever presented with an appropriate set \( P \), the multal adjective many selects those elements of \( P \) which are sums of a large number of atoms. The paucal adjective few, on the other hand, selects those elements of \( P \) which are sums of a small number of atoms.

It should be pointed out that what should count as large or small is left open, but must depend, at least in part, on \( \mu \) and on what \( P \) is. Thus some \( \mu(x) = 5 \) may count as large in \( P \) if \( P \) is the domain of outer planets, but as negligibly small if \( P \) is the domain of cities. Yet, since what counts as large or small has been left open, a speaker may choose to regard five cities as many. He will be committed, however, to the awkward view that five out of thousands constitute many.

Incidentally, notice that contextual considerations will ultimately play a role in determining whether or not \( \mu(x) \) is large or small in \( P \). Consider for instance a class of thirty students. Suppose ten of these students received an A for the course. Is Many students of the class received an A true? It would seem so. But suppose now that only ten students of the class are righthanded. Can we say that Many students of the class are righthanded? It would seem that we cannot. Ten out of thirty counts as many in the context of receiving an A but not in the context of being righthanded. Notice that nothing changes if the students who received an A were, precisely, the righthanded students. The very same group of students may count as many for some purposes but not for others (cf. Westerståhl 1985).

In fact, the very question of whether or not \( \mu(x) \) is large or small in \( P \) does not seem to admit of a definite, yes or no, answer (cf. McCawley 1981, 428). Consequently, membership in the set of \( Ps \) which are many seems to be a matter of degree. If so, the sets in (24) would be fuzzy in the sense of Zadeh (1965). Finally, notice that many and few seem to admit of senses other than the ones described in (24). Consider for instance a sentence like Many Scandinavians have won the Nobel Prize in Literature. This sentence seems to have two readings: one in which it

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15 The proportionality of many and few may thus depend on the cardinality of sets other than the ones contributed by the nouns they combine with. The readings of many and few are therefore 'loosely proportional' in the sense of Partee (1988, §3), who notes the difficulty of distinguishing such readings from their cardinal counterparts.
is clearly false, since fourteen out of all the millions of Scandinavians should not count as many; the other in which it is true, since fourteen Nobel prizes in Literature constitutes quite a large share thereof. The analysis in (24) describes only the former of these senses, as the latter is synonymous with Many winners of the Nobel prize in Literature have been Scandinavian (cf. again Westerståhl 1985).

To illustrate the external semantics of multal and paucal adjectives in (24), we shall call forth a second universal semantic principle. It is the following:

(25) The Functional Principle

Whenever an expression which denotes a function combines with an expression which denotes a potential argument for that function, the denotation of the resulting expression is the actual application of the function to the potential argument.

As with the Intersection Principle, the Functional Principle is motivated independently of the matters at hand, as it governs the interpretation of a variety of expressions whose parts can combine meaningfully in just one way. In any event, we again regard this principle as a markedness convention which will be automatically overruled by any specific stipulations which are incompatible with it.

Equipped with the conditions in (24) and the principle in (25), the external semantics of paucal and multal adjectives can be illustrated by the interpretations in (26).

(26) a. `[many rings] = [many][[rings]] = [many](R*) = \{x \in R*: \mu(x) \text{ is large}\}.

b. `few rings = [few][[rings]] = [few](R*) = \{x \in R*: \mu(x) \text{ is small}\}.

It should not escape the reader that multal and paucal adjectives combine with nominals to form nominals. See Chapter 5, Section 16, for proposals concerning the noun phrases constituted by the resulting nominals.

11 Some Assumptions

In order to address the interaction of determination and countability we must first be precise about the denotation of sentences and noun phrases. To do so let us once again follow Montague (1973) and assume, in

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16 The Functional Principle also amounts to an instance of what Klein & Sag (1985) have called type driven translation. This principle will be invoked repeatedly in the sequel.
effect, that verb phrases denote *sets of individuals* of the model while noun phrases denote *sets of sets of individuals* of the model. As a consequence of these two assumptions, it will follow that a sentence constituted by a noun phrase and a verb phrase can be taken to be true just in case the set associated to the latter is an element of the set of sets associated to the former. More formally, we assume the conditions in (27) and (28), where $E$ stands for the universe.\(^{17}\)

(27)  
\begin{align*}
&\text{a. Let $VP$ be a verb phrase. If $[[VP]]$ is defined, then it is a subset of $E$.} \\
&\text{b. Let $NP$ be a noun phrase. If $[[NP]]$ is defined, then it is a set of subsets of $E$.}
\end{align*}

(28)  
\begin{align*}
&\text{Let $S$ be a sentence constituted by an $NP$ and a $VP$. If $[[NP]]$ and $[[VP]]$ are defined, $S$ is true if $[[VP]] \in [[NP]]$ and false if $[[VP]] \notin [[NP]]$.}
\end{align*}

It should be noticed that (28) does not assert that every sentence must be either true or false. In fact, everything else being the same, a sentence will be neither true nor false whenever the denotation of its subject or that of its predicate is undefined. As will be seen in the sequel, we will want to reserve the option of allowing sentences whose denotation is in fact undefined. Naturally, such sentences will be nonpropositional: they will be neither true nor false.\(^{18}\)

12 Universal Determiners

Having availed ourselves of the conditions in (27) and (28), we may now turn to the universal determiners *every* and *all*, which we propose to interpret as shown in (29). As usual, we proceed here relative to a model which provides a universe $E$. As might be expected, an atomic $P \subseteq E$ is a set of atoms of the universe.

\(^{17}\) We note that the denotations in (27) and (28) should be formulated not in terms of sets, but rather in terms of the characteristic functions of these sets. When thus formulated, (28) becomes an instance of the Functional Principle, and need not be independently stipulated (see Cooper (1983, 21ff) for an illuminating discussion of the general semantics of noun phrases and sentences). Alternatively, we could let noun phrases denote *sets of sets of individuals* and verb phrases denote *sets of sets of sets of individuals*. Then sentences would be true if their subjects denoted elements of the sets denoted by their predicates (see Dowty and Brodie (1984, 76) and the references cited therein).

\(^{18}\) Alternatively, we might regard all sentences as propositional relative to a truth value set with three members: the true, the false and the undefined.
(29) **Universal Determiners**

a. The determiner *every* denotes a function which assigns, to each atomic $P \subseteq E$, the family $\{X \subseteq E: P \subseteq X\}$.

b. The determiner *all* denotes a function which assigns, to each $P \subseteq E$, the family $\{X \subseteq E: P \subseteq X\}$.

As can be readily seen, universal determiners are similar in that they identify the supersets (relative to the universe) of a given set. They are different, however, as to the sets they can identify the supersets of. *Every* identifies the supersets of sets of atoms of the universe while *all* identifies the supersets of arbitrary sets of individuals.

To illustrate the internal semantics of universally quantified noun phrases, let us consider the interpretations in (30) and (31), which incorporate assumptions, interpretations, and notational conventions which should be by now familiar.

(30) $\text{[[every ring]]} = \text{[[every]]([[ring]])} = \text{[[every]](R)} = \{X \subseteq E: R \subseteq X\}
\text{[[all rings]]} = \text{[[all]]([[rings]])} = \text{[[all]](R*)} = \{X \subseteq E: R* \subseteq X\}

(31) $\text{[[every thing]]} = \text{[[every]]([[thing]])} = \text{[[every]](A)} = \{X \subseteq E: A \subseteq X\}
\text{[[all things]]} = \text{[[all]]([[things]])} = \text{[[all]](A*)} = \{X \subseteq E: A* \subseteq X\}

To illustrate the external semantics of universally quantified noun phrases, let us consider now the sentence in (32).

(32) Every ring is expensive.

Let us moreover consider a model whose universe is as diagramed in (1) and which interprets the noun *ring* as the set $\{a, b\}$ enclosed in Figure 5 above. Let us finally suppose that the set of entities which are expensive in this universe is $\{b, c, b+c\}$, which is the set that has been represented in Figure 9:

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19 Formally, $A$ is a superset of $B$ (relative to $U$) if and only if $A \supseteq B$ (and $A \subseteq U$).
It follows from (28) and (29a) that sentence (32) will be true if \( \{b, c, b+c\} \) is a superset of \( \{a, b\} \) and false if it is not. But \( \{b, c, b+c\} \) is clearly not a superset of \( \{a, b\} \). Hence sentence (32) is false relative to the model in question. Moreover, if \([\text{is expensive}]\) is not a superset of \([\text{ring}]\) in this model, then it will certainly not be a superset of \([\text{rings}]\), the more inclusive set represented in Figure 6 above. And more generally, whenever rings is taken in its general sense, then the falsity of (32) will entail the falsity of (33).

(33) All rings are expensive.

To further illustrate the external semantics of universally quantified noun phrases we will turn to the sentence in (34).

(34) All rings look alike.

Let us assume that look alike denotes a function which maps the domain of cars onto the set of cars which look alike, the domain of rings onto the set of rings which look alike, the domain of dogs onto the set of dogs which look alike—and so on. Now, what domain should the function contributed by look alike apply to in a sentence like (34)? Presumably to the domain of rings. Let us generalize this and assume that the function contributed by a predicate like look alike always applies to the domain denoted by the stem of its subject noun. It follows that the function contributed by look alike in (34) will apply to the domain denoted by the stem ring contained therein. (34) will thus be interpreted along the lines of all rings are rings which look alike—a result which intuition will corroborate.

It might be thought that the foregoing requires the abandonment of the Principle of Compositionality. As we see it, however, it does not, for it might be argued that the function denoted by look alike does not have 'textual access' to the domain denoted by the stem ring; its access thereto is only 'contextual'. Although an adequate discussion of the specifics of this proposal would take us too far afield, the idea we have in mind is that subject stems contribute domains to the context or discourse—subjects, after all, tend to contribute topics. Such domains are then available to functions which, like look alike, would otherwise be lacking their arguments. Similar appeals by predicates to subject nouns are not unheard of (cf. Kamp 1975, 123).

Be that as it may, let us assume now that only mereologically complex individuals can look alike. To be more precise, let us say that look alike denotes a function which assigns, to each domain in the universe, a set of nonatoms of that domain—namely those which look alike.
Now, if we are given (28) and (29), the sentence in (34) will make the assertion in (35).

(35) \[ [[rings]] \subseteq [[look alike]]([[ring-]]) \]

Let us say now that the universe is as diagramed in (1). Let us moreover say that the plural on \textit{rings} was taken in its specific meaning\textsuperscript{20} so that \textit{rings} can denote the set \{a+b\} of Figure 7. Let us say, finally, that the set of rings which look alike in this universe is (again) the set \{a+b\} of Figure 7. It follows that the set of rings which look alike is an (improper) superset of the set of rings, and (34) is therefore true.

Naturally, if the plural on \textit{rings} was taken in its general meaning, then [[\textit{rings}]] will contain atomic rings (we assumed that the plural was the unmarked member of the opposition of number). But [[\textit{look alike}]]([[\textit{ring-}]]) cannot contain any atomic rings (we assumed that only mereologically complex individuals may look alike). As a consequence of this, the set of rings which look alike could never be a superset of a set of rings, and (34) would therefore have to be false. The specific reading of \textit{rings} is thus forced by context in (34) if contradctoriness is to be avoided.

Now, we have assumed that \textit{look alike} denotes a function which selects a set of nonatoms from each domain in the universe. But it seems reasonable to assume that the same holds for \textit{looks alike}, so that the singular inflection on \textit{looks} in (36a) is a syntactic reflex of no semantic consequence. If this is so, then (36b) would be the assertion of (36a)—at least given (28) and (29).

(36) a. ?Every ring looks alike.

b. \[ [[\textit{ring}]] \subseteq [[\textit{looks alike}]]([[\textit{ring-}]]) \]

But notice now that [[\textit{looks alike}]]([[\textit{ring-}]]) must be a set of nonatoms while [[\textit{ring}]] must be a set of atoms. This means that (36b) can never be true—but then neither can (36a), which is therefore ill formed.

We conclude this section with the observation that the inclusion in (37) will obtain for every atomic subset \(P\) of the universe.

(37) \[ [[\textit{all}]](P*) \subseteq [[\textit{every}]](P) \]

For, if some set is a superset of \(P^*\), then it will certainly be a superset of the less inclusive set \(P\) as well. Naturally, the converse of this inclusion will not obtain for all values of \(P\).

\textsuperscript{20} The process whereby the plural acquires a specific meaning was discussed in Section 6 above.
13 Existential Determiners

Let us turn now to the unstressed existential determiners *a* and *some*, which we propose to interpret as shown in (38).\(^{21}\) We once again proceed relative to a model which provides a universe \(E\). As before, an atomic \(P \subseteq E\) is a set of atoms of the universe.

(38) **Existential Determiners**

- a. The determiner *a* denotes the function which assigns, to each atomic \(P \subseteq E\), the family \(\{X \subseteq E: P \cap X \neq \emptyset\}\).
- b. The determiner *some* denotes the function which assigns, to each \(P \subseteq E\) the family \(\{X \subseteq E: P \cap X \neq \emptyset\}\).

As can be readily seen, existential determiners are similar in that they identify the conjoints (relative to the universe) of a given set.\(^{22}\) They are different, however, as to the sets they can identify the conjoints of. The determiner *a* identifies the conjoints of sets of atoms of the universe, while the determiner *some* identifies the conjoints of arbitrary sets of individuals.

To illustrate the internal semantics of existentially quantified noun phrases, let us consider the interpretations in (39) and (40), which again incorporate familiar assumptions, interpretations, and notational conventions.

(39) \[
\begin{align*}
\llbracket a \text{ ring} \rrbracket &= \llbracket a \rrbracket(\llbracket \text{ring} \rrbracket) = \llbracket a \rrbracket(R) = \{X \subseteq E: R \cap X \neq \emptyset\} \\
\llbracket \text{some rings} \rrbracket &= \llbracket \text{some} \rrbracket(\llbracket \text{rings} \rrbracket) = \llbracket \text{some} \rrbracket(R^*) \\
&= \{X \subseteq E: R^* \cap X \neq \emptyset\}
\end{align*}
\]

(40) \[
\begin{align*}
\llbracket a \text{ thing} \rrbracket &= \llbracket a \rrbracket(\llbracket \text{thing} \rrbracket) = \llbracket a \rrbracket(A) = \{X \subseteq E: A \cap X \neq \emptyset\} \\
\llbracket \text{some things} \rrbracket &= \llbracket \text{some} \rrbracket(\llbracket \text{things} \rrbracket) = \llbracket \text{some} \rrbracket(A^*) \\
&= \{X \subseteq E: A^* \cap X \neq \emptyset\}
\end{align*}
\]

To illustrate the external semantics of existentially quantified noun phrases, let us consider now the sentence in (41).

(41) A ring is expensive.

Let us once again consider a model whose universe is as diagramed in (1) and which interprets the noun *ring* as the set enclosed in Figure 5 above. Let us suppose, finally, that the set of entities which are expensive in this universe is the set which has been represented in Figure 9 above. It follows from (28) and (38a) that the sentence in (41) will be true if \(\{b, c, b+c\}\) is a conjoint of \(\{a, b\}\) and false if it is not.

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\(^{21}\) We use *some* (read /sǝm/) to refer to the unstressed counterpart of *sόme* (read /sǝm/). See Quirk et al. (1985, §5.14).

\(^{22}\) A is a conjoint of B (relative to U) if and only if \(A \cap B \neq \emptyset\) (and \(A \subseteq U\)).
But \( \{b, c, b+c\} \) clearly overlaps with \( \{a, b\} \). Hence (41) is true relative to the model in question. Moreover, if \([is \text{ expensive}]\) is a conjoint of \([ring]\) in this model, then it will certainly be a conjoint of \([\text{rings}]\), the more inclusive set represented in Figure 6 above. And more generally, whenever rings is taken in its general sense, the truth of (41) will entail the truth of (42).

(42) Some rings are expensive.

To further illustrate the external semantics of existentially quantified noun phrases, we will turn to the sentence in (43a).

(43) a. Some rings look alike.
   b. \([\text{rings} \cap \text{look alike} \cap \text{rings}^*] \neq \emptyset\)

It will be noticed that the truth value of this sentence will depend on the model against which it is interpreted. For notice that (43a) will be true only in models in which the set of rings which look alike properly intersects the set denoted by rings. This is indicated in (43b). But suppose now that rings denotes the set \( \{a, b, a+b\} \) enclosed in Figure 6 above. Suppose further that the set of rings which look alike is the set \( \{a+b\} \) of Figure 7. Since these two sets overlap, (43a) would be true relative to this model. But consider a model in which the set of rings which look alike is empty. Relative to this model, the set of rings which look alike does not properly intersect the set denoted by rings. (43a) would thus be false relative to this model.

The truth of (43a) is thus dependent upon the models against which it is evaluated. But consider now the sentence in (44a), whose truth conditional contribution is as indicated in (44b).

(44) a. A ring looks alike.
   b. \([\text{ring} \cap \text{looks alike} \cap \text{ring}^*] \neq \emptyset\)

The truth value of this sentence does not depend on the model against which it is interpreted. To see this recall that \([\text{looks alike} \cap \text{ring}^*]\) is a set of nonatomic in every model. Recall also that \([\text{ring}]\) is a set of atoms in every model. But then there is no model in which these two sets can have a proper intersection. This means that (44a) will be false in every model (it will thus be contradictory). As a consequence of this, (44a) will be ill formed.

We wish to close this section with two observations. The first is that the inclusion in (45) will obtain for every atomic subset \( P \) of the universe.

(45) \([a](P) \leq [\text{some}](P^*)\)
58 Linguistic Individuals

For, if some set is a conjoint of $P$, then it will certainly be a conjoint of the more inclusive set $P^*$ as well. Naturally, the converse of this inclusion will not obtain for all values of $P$.

Our second observation pertains the inclusions in (46), where $P$ is an atomic, nonempty, subset of the universe.

(46) a. $[[\text{every}]](P) \subseteq [[\text{all}]](P)$
   b. $[[\text{all}]](P^*) \subseteq [[\text{some}]](P^*)$

For every superset of a nonempty set is a conjoint thereof (though not conversely).

14 Nonexistential Determiners

Let us turn now to the nonexistential determiner $no$. We propose to interpret it as shown in (47). Here too we proceed relative to a model which provides a universe $E$.

(47) The Nonexistential Determiner

The determiner $no$ denotes the function which assigns, to each $P \subseteq E$, the family $\{X \subseteq E: P \cap X = \emptyset\}$.

As can be readily seen, the nonexistential determiner identifies the disjoints (relative to the universe) of a given set.\(^{23}\)

To illustrate the internal semantics of the nonexistentially quantified noun phrases, let us consider the interpretations in (48) and (49), which proceed relative to the usual assumptions.

(48) $[[\text{no ring}]] = [[\text{no}]]([[\text{ring}]]) = [[\text{no}]](R) = \{X \subseteq E: R \cap X = \emptyset\}$

(49) $[[\text{no things}]] = [[\text{no}]]([[\text{things}]]) = [[\text{no}]](A^*) = \{X \subseteq E: A^* \cap X = \emptyset\}$

To illustrate the external semantics of nonexistentially quantified noun phrases, let us consider now the sentence in (50).

(50) No ring is expensive.

Let us once again consider a model whose universe is as diagramed in (1) and which interprets the noun ring as the set represented in Figure 5 above. Let us suppose, finally, that the set of entities which are expensive in this universe is the set which has been represented in Figure 9 above. It follows from (28) and (47) that the sentence in (50)

\(^{23}\) Formally, $A$ is a disjoint of $B$ (relative to $U$) if and only if $A \cap B = \emptyset$ (and $A \subseteq U$).
will be true if \( \{b, c, b+c\} \) is a disjoint of \( \{a, b\} \) and false if it is not. But \( \{b, c, b+c\} \) and \( \{a, b\} \) are clearly not disjoint. Hence (50) is false relative to the model in question. Moreover, if \([is expensive]\) is not a disjoint of \([ring]\) in this model, then it will certainly not be a disjoint of \([rings]\), the more inclusive set represented in Figure 6 above. And more generally, whenever rings is taken in its general sense, then the falsity of (50) will entail the falsity of (51).

(51) No rings are expensive.

But let us consider a situation in which both (50) and (51) are true. Interestingly, this state of affairs is more naturally described by (51) than by (50). To account for this fact we can invoke the suggestion, made to me by James McCawley, that no is used with a singular noun when there is a presupposition that there would be only one if there were any (and with a plural noun otherwise). Clearer evidence for this suggestion is provided by the preference of (a) over (b) in both (52) and (53).

(52) a. No Pope has been elected yet.
    b. No Popes have been elected yet.

(53) a. This show has no viewers.
    b. This show has no viewer.

The preference of (51) over (50) would thus be due to the presupposition that if there were any expensive rings, then there would probably be more than one.

To further illustrate the external semantics of nonexistential determiners, we will turn to the sentence in (54a).

(54) a. No rings look alike.
    b. \([rings] \cap [look alike][[ring-]] = \emptyset\)

It will be noticed that the truth value of this sentence will depend on the model against which it is interpreted. For, given the semantics of look alike presented above, (54a) will be true only in models in which the set of rings which look alike is disjoint from the set denoted by rings. This is indicated in (54b). But suppose now that rings denotes the set \( \{a, b, a+b\} \) enclosed in Figure 6 above. Suppose further that the set of rings which look alike is empty. Since these two sets are disjoint, (54a) would be true in this model. But consider a model in which the set of rings which look alike is the set \( \{a+b\} \) of Figure 7. Relative to this model, the set of rings which look alike is not disjoint from the set denoted by rings. (54a) would thus be false relative to this model.

The truth of (54a) is thus dependent upon the models against which it is evaluated. But consider now the sentence in (55a), whose truth conditional contribution is as indicated in (55b).
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(55) a. No ring looks alike.
      b. \([\text{\text{ring}}] \cap [\text{\text{looks alike}}]([\text{\text{ring}}]) = \emptyset\)

The truth value of this sentence does not depend on the model against which it is interpreted. To see this recall that \([\text{\text{looks alike}}]([\text{\text{ring}}])\) is a set of nonatomic in every model. Recall also that \([\text{\text{ring}}]\) is a set of atoms in every model. This means that (55a) will be true in every model (it will thus be valid). The extreme un informativeness of validity can then account for the peculiarity of (55a).

We conclude this section with the observations that the equalities in (56) and (57) will obtain for every atomic subset \(P\) of the universe and for \(\emptyset(E)\) the set of all subsets of the universe.

(56) a. \(\llbracket a \rrbracket(P) \cap \llbracket \text{no} \rrbracket(P) = \emptyset\)
      b. \(\llbracket a \rrbracket(P) \cup \llbracket \text{no} \rrbracket(P) = \emptyset(E)\)

(57) a. \(\llbracket \text{some} \rrbracket(P^*) \cap \llbracket \text{no} \rrbracket(P^*) = \emptyset\)
      b. \(\llbracket \text{some} \rrbracket(P^*) \cup \llbracket \text{no} \rrbracket(P^*) = \emptyset(E)\)

(the set of conjoints of any set is mutually exclusive and jointly exhaustive with the set of disjoints of that set).

15 Dual Determiners

Let us turn now to the determiners \textit{both} and \textit{neither}, which we propose to interpret as shown in (58). Here we again proceed relative to a model which provides a universe \(E\). As before, an atomic subset \(P\) of the universe is a set of atoms of the universe.

(58) \textbf{Dual Determiners}

a. The determiner \textit{both} denotes a function which assigns, to each \(P^*\), the family \(\{X \subseteq E : P \subseteq X\}\), where \(P\) is an atomic subset of the universe and has exactly two elements.

b. The determiner \textit{neither} denotes a function which assigns, to each \(P\), the family \(\{X \subseteq E : P \cap X = \emptyset\}\), where \(P\) is an atomic subset of the universe and has exactly two elements.

To illustrate the semantic contribution of these determiners, let us consider a model whose universe contains two and only two rings. The interpretations of \textit{both rings} and \textit{neither ring} relative to such a model will be the ones in (59), where we make the usual assumptions.

(59) a. \([\text{both rings}] = [\text{both}](\llbracket \text{rings} \rrbracket) = [\text{both}](R^*)
               = \{X \subseteq E : R \subseteq X\} \)

---

24 We stop short of regarding the sentence as semantically illformed. Valid sentences would seem to be best treated as semantically wellformed sentences which are pragmatically illformed.
b. $[[\text{neither ring}]] = [[\text{neither}]]([[\text{ring}]]) = [[\text{neither}]](R) = \{X \subseteq E : R \cap X = \emptyset\}$

And if in this model the set of atomic rings is as indicated in Figure 5 while the set of expensive entities is as indicated in Figure 9, the two sentences in (60) will be false, as $[[\text{expensive}]]$ is neither a superset nor a disjoint of $[[\text{ring}]]$.

(60) a. Both rings are expensive.
    b. Neither ring is expensive.

Finally, notice that both identifies the supersets of particular sets while neither identifies the disjoints of particular sets. This means that the equalities in (61) will hold in any model which abides by (29a), (47), and (58).

(61) a. $[[\text{both}]](P^*) = [[\text{every}]](P)$ for any atomic $P \subseteq E$ such that $|P| = 2$.
    b. $[[\text{neither}]](P) = [[\text{no}]](P)$ for any atomic $P \subseteq E$ such that $|P| = 2$.

It follows that we can give a unified account of the semantic illformedness in (36a) and (62a) and that we can give a unified account of the pragmatic illformedness in (55a) and (62b).

(62) a. ?Both rings look alike.
    b. Neither ring looks alike.

16 The Definite Article: Descriptive Uses

Let us begin by recalling that the mereological postulate of completeness ensures that every nonempty subset of a mereology has a unique sum. But there is nothing to ensure that such a sum will be an element of that subset. If it is, we will say that this sum is the greatest element of the set (or that the set has a greatest element); if it is not, then we will say that the set does not have a greatest element.

To illustrate, let us say that our universe of discourse is as diagramed in (1) above. Consider now $\{a, b\}$ and $\{a, b, a+b\}$, the sets which have been enclosed in Figures 5 and 6 above. Note that $\{a, b\}$ and $\{a, b, a+b\}$ have the same sum, namely $a+b$. Such a sum, however, is only an element of $\{a, b, a+b\}$; it is not an element of $\{a, b\}$. We can therefore say that $\{a, b, a+b\}$ has a greatest element, whereas $\{a, b\}$ does not. In other words, $a+b$ is the greatest element of $\{a, b, a+b\}$ while it is only the sum of $\{a, b\}$.

Now, it is easy to see that if a set has a greatest element, then it has at most one (every greatest element of a set is a sum of all elements of the set and sums are unique). Hence we can define a
function which maps each subset of a mereological field which has a greatest element into that greatest element. We will henceforth say, somewhat misleadingly, that this is $\Gamma$, the 'greatest function' of the mereology.

Having defined the greatest function $\Gamma$ of a mereology, we can now incorporate the remarkable theory of definite descriptions advanced in Sharvy (1980) and straightforwardly interpret the definite article as shown in (63).

(63) The Definite Article

The determiner *the* denotes the greatest function of the universe.

To illustrate, let us consider the interpretation of *the rings* relative to a model which abides by conditions (25) and (63) and provides a universe which is as diagramed in (1). If the set of rings in this universe is our set $R = \{a, b\}$ of Figure 5, then

(64) $[[\text{the rings}]] = [[\text{the}]]([[\text{rings}]]) = \Gamma(R^*) = a+b$

Let us notice now that $\Gamma(F)$ will be undefined whenever $F$ is the empty subset of the universe. It follows that the denotation of a phrase constituted by the definite article and a 'nondenoting' noun will be undefined. Casting this observation in positive terms, we arrive at a

(65) Presupposition of Existence

Let $NP$ be a noun phrase immediately constituted by the definite article and a nominal. If $[[NP]]$ is defined, then the nominal denotes a nonempty subset of the universe.

To illustrate, let us attempt to interpret *the two rings* and *the three rings* relative to a model whose set of rings contains exactly two members (and follows the usual assumptions). Consider first *the two rings*. This noun phrase will denote the sum $a+b$ of the two rings of the model. As predicted by (65), its nominal *two rings* will denote a nonempty subset of the universe. This subset is $\{a+b\}$. But consider *the three rings* instead. Its nominal *three rings* will denote the empty set in our paradigmatic universe. And as (65) would lead us to expect, the interpretation of the complete noun phrase is indeed undefined.25

25 It should be borne in mind that the presupposition of existence in (65) asserts only that some nominals denote nonempty subsets of the universe. It does not assert (nor does it imply) that such subsets are furthermore contained in the denotation of $\exists$. If there can be elements of the universe of discourse which are not elements of $\exists(\text{exists})$ (as urged in Parsons 1980), then we can account for the fact that (i) can be asserted without contradiction

(i) The philosopher's stone does not exist.
To further illustrate the force of (65), let us consider the interpretation of the noun phrase *the king of France*. Suppose for the sake of argument that this interpretation proceeds relative to a model whose set of individuals contains no king of France. (65) then predicts that the interpretation of the celebrated noun phrase is undefined. Following the general assessment of the data, we take such predictions to be correct.26

It is interesting to note that a denotation for vacuous definite descriptions was defined in Carnap (1947, §8). Following earlier suggestions by Frege, he assigned these phrases a 'null individual' which would lack any (distinctive) properties. This individual was conceived of as the least element in a partial order. It should be clear that this proposal translates only into a degenerate mereological setting, since null individuals will exist only in universes with exactly one element.

Consider now the set of *atoms* contained in some domain of the universe. This set will itself contain a greatest element just in case it contains exactly one element. It follows from (25), (63) and the semantics of singular nouns that definite noun phrases (in the singular) have the presupposition of uniqueness stated in (66).

(66) **Presupposition of Uniqueness**

Let $NP$ be a noun phrase immediately constituted by the definite article and a singular noun. If $[NP]$ is defined, then the noun denotes a singleton subset of the universe.

Thus, if $[[\text{ring}]] = \{a, b\}$, then the denotation of the singular noun phrase *the ring* is undefined. If we again follow the consensus, this is the correct prediction to make.27

Now, following Karttunen (1976), McCawley (1981, §9.6) argues that definite descriptions denote unique individuals only relative to a 'contextual domain', not relative to an entire universe of discourse. Contextual domains are structured sets of entities which are built as discourse proceeds. They have been developed in Kamp (1981) and also in Heim (1982), who ultimately denies, however, that definite descriptions have a presupposition of uniqueness. McCawley’s proposal can be easily incorporated into the present framework simply by stipulating that the arguments of $\Gamma$ be subsets of the desired contextual domain. See also Westerståhl (1984, 59).

—and the fact that (i) can be asserted without contradiction should not lead us to question the correctness of (65).

26 See Neale (1990), however, for an alternative view.

27 See Chapter 4, Sections 16 and 17, for our account of the generic use of the *ring*. 
It should be noticed that the preceding presupposition of uniqueness should not be extended to the (semantic) plural. Consider for instance a plural noun phrase like *the rings*. As any plural noun, *rings* denotes a set which contains a greatest element (if it denotes anything at all). Hence a definite noun phrase in the plural will always be defined (if the denotation of its head nominal is not the empty set). That this should be as desired is borne out by the obvious fact that the denotation of *the rings* is defined even if the nominal phrase *rings* does not denote a singleton.

But let us return to the singular uses of the definite article. As the reader will no doubt have realized, (63) entails the essentials of the celebrated analysis of the definite article originally proposed early in this century by Bertrand Russell. Thus, when taken in conjunction with (28) and other independently motivated interpretations, (63) entails that a sentence like *The father of Charles II was executed* is true (relative to a model) if and only if there is an individual (in the model) who begat Charles II, there is only one such individual, and this individual was executed.

But beyond this, the claim being made by (63) is that the presuppositions of existence and uniqueness which inhere in the Russellian semantics of *the* arise not only as a matter of fact, but rather as a matter of principle: both follow jointly from the requirement that the denotation of the definite article be the greatest function of the universe. Moreover, the analysis proposed in (63) extends to plural definite descriptions. (63) thus differs from the Russellian proposal in that it provides a unified account of (countable) definite descriptions; the semantics of the definite article applies regardless of singularity.

But (63) departs from the Russellian analysis in another important respect: it leaves open the issue of the denotation of sentences whose presuppositions of existence and uniqueness fail. True, our sentential semantics (28) entails that if the subject or the predicate of a sentence has an undefined denotation, the sentence as a whole will also have an undefined denotation (everything else being the same). Thus, (28) entails that the denotation of the much discussed sentence *The king of France is bald* is undefined whenever the denotation of its subject is undefined. But, crucially, the indefiniteness of the denotation of such a

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28 See Russell (1905).
29 This now widely held claim was made originally by Strawson (1950) and Geach (1950). Naturally, a suitable logic with three values could combine with the analysis advocated here to make the claim that the truth value of *The king of France is bald* is defined: it would be the truth value which is neither the true nor the false. See Note 18 above.
sentence does not follow from (63) alone. For it is only when (63) is taken in conjunction with (28) that the denotation of the celebrated sentence will be predictably undefined.

We conclude this section by discussing the theory of definite descriptions advanced in Sharvy (1980). When stripped to its essentials, this theory states that any noun phrase of the form the P must be assigned an expression which expands to (67).

\[(67) \exists x[P x \land \forall y[P y \rightarrow y \leq x]]\]

Notice that since Sharvy interprets '≤' as the relation 'is part of', he would gloss (67) as '(there exists) a P every P is part of'. This means that a definite description of the form the P would in effect assert the existence of a greatest element in the set denoted by P (while remaining neutral as to the interpretation of the definite article proper).

As far as I can see, there are three nonnegligible advantages of (63) over the proposals of Sharvy. The first is that the definite article is actually provided by us with an interpretation. The second is that a definite description is not taken to assert the existence of a greatest element, but only to presuppose it (this means that any sentence which contains an improper definite description will be nonpropositional for us but false for Sharvy). The third is that we have provided definite descriptions with a 'textual' rather than a 'contextual' interpretation (for us, definite descriptions may actually denote, whereas for Sharvy, only the sentences which contain definite descriptions actually denote). It does not escape us, however, that these three differences have less to do with Sharvy's proposals than with the Russellian framework in which they were embedded.

The present proposals about the semantics of definite descriptions also contrast with the one found in Link (1983), where the definite article (or rather the iota operator) is used to define the sum operator. It should be pointed out, however, that in subsequent work, Link (1987, 178n) observes that whenever P is a singleton, \(\Sigma(P)\) and \(\iota P\) denote the same thing, so that "one could take this as a definition of \(\iota\) in terms of \(\Sigma\). This is perhaps more natural than doing it the other way around, as in Link 1983." See also Blau (1981) and Krifka (1987, 11).

17 Syntactic Denotations

The careful reader will have noticed that every noun phrase constituted by a definite article and a nominal will denote an element of the universe (an entity whose logical type is zero) whereas universally, existentially, and nonexistentially quantified noun phrases will denote sets of sets of individuals of the universe (entities whose logical type is
two). But consider now the semantics of sentences. As will be recalled, a sentence composed of a noun phrase and a verb phrase was taken to assert that the set (of individuals) denoted by the verb phrase was an element of the set of sets (of individuals) denoted by the noun phrase. But what if the noun phrase was definite? It would not denote the prerequisite set of sets of individuals. In fact, it would not even denote a set of individuals.

Naturally, a unified semantics for sentences could be abandoned and sentences with definite subjects could be interpreted differently than sentences with quantified subjects. Instead of doing this, we will follow Barwise and Cooper (1981) and distinguish between the morphological and the syntactic denotations of the definite article. To do so, let us assume that the interpretation in (63) provides a morphological denotation for the definite article. It indicates what this form can mean as a word. This interpretation would thus leave open what the syntactic denotation for this form can be. It does not specify what this form can denote as a phrase.

Now, let us assume for the sake of definiteness that any adequate grammar of English will contain the lexical insertion in (68) and that this lexical insertion has the semantic counterpart in (69).

(68) \( DET \rightarrow \text{the} \)

(69) **The Definite Article (Syntactic Denotation)**

\( DET \) denotes the function which assigns, to each \( F \subseteq E \) which has a greatest element, the family \( \{ X \subseteq E : [[\text{the}]](F) \in X \} \).

Thus, the syntactic denotation of the definite article is a function which assigns, to each suitable subset \( F \) of the universe, the set of sets which have \( F(F) \) as member. Definite noun phrases may thus denote sets of sets, and a unified semantics of sentences is thereby attained.\(^3\)

Having availed ourselves of a syntactic denotation for the definite article, we may now account for three fundamental properties of definite descriptions. The first pertains to the relations between \( \text{the} \) and \( \text{every} \) predicted by our proposals. These relations are given by (70) and follow from the fact that both \( \text{the} \) and \( \text{every} \) have superset functors as their (syntactic) denotations.

(70) If \( N \) is a singular noun then \( [[\text{the} \, N]] = [[\text{every} \, N]] \) whenever both terms of the equation are defined.

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\(^3\) So the definite article in effect exhibits an ambiguity in logical type, as suggested in Partee (1987, 124).
To illustrate, let us say that there is but one expensive ring in our favored model. It is the atom $b$. Now,

\[
[[\text{the expensive ring}]] = [[\text{the}]]([[\text{expensive ring}}]) = \{X \subseteq E: \Gamma((b)) \in X\} \\
= \{X \subseteq E: b \in X\} = \{X \subseteq E: \{b\} \subseteq X\} \\
= [[\text{every}]]([[\text{expensive ring}}]) \\
= [[\text{every expensive ring}]]
\]

And this synonymy seems to follow intuition closely. As has been pointed out in the literature (cf. e.g., Barwise and Cooper 1981, 169), singular definite descriptions have universal force.

A second property which follows from the proposals we have advanced pertains to the equivalences in (72). They obtain relative to any model in which the denotations contained therein are defined. Since every set $P$ is such that $P \supseteq P$, the proposals we have advanced will entail that the sentences in (71) are true relative to any model in which their denotations are defined.\footnote{The sentences in (71) are instances of what E.W. Beth called 'Plato's Principle'. See Barth (1974, 39).}

(71) The ring is a ring.
The rings are rings.

(72) $[[\text{the ring is a ring}]] \leftrightarrow [[\text{ring}]] \subseteq [[\text{the ring}]]$ $\leftrightarrow [[\text{ring}]] \supseteq [[\text{ring}]]$

$[[\text{the rings are rings}]] \leftrightarrow [[\text{rings}]] \subseteq [[\text{the rings}]]$ $\leftrightarrow [[\text{rings}]] \supseteq [[\text{rings}]]$

A third property of definite descriptions pertains to the fact that noun phrases like the two rings and both rings turn out to be interestingly nonsynonymous under the present account. Consider first both rings, which has been seen to denote as shown in (73a)—at least provided $|R| = 2$. Consider next the two rings. It will be seen that its denotation is as shown in (73b)—at least under the same proviso.

(73) a. $[[\text{both rings}]] = \{X \subseteq E: R \subseteq X\}$

b. $[[\text{the two rings}]] = \{X \subseteq E: \Gamma(2R) \in X\} = \{X \subseteq E: \Sigma(R) \subseteq X\} = \{X \subseteq E: \{\Sigma(R)\} \subseteq X\}$

Thus, if $R$ is the set of Figure 5, then both rings denotes the superset of \{a, b\} while the two rings denotes the superset of \{a+b\}. This subtle difference is borne out by the facts. Consider for instance the contrast between (74a) and (74b).

(74) a. The two rings look alike.

b. ?Both rings look alike.

(74a) asserts that the set of rings which look alike is a superset of a set which contains a sum of two rings. But such a sum is a nonatomic
entity. Hence, if the entities which look alike are also nonatomic, (74a) may be true. But (74b) asserts that the set of rings which look alike is a superset of a set with atomic rings. Hence, if the entities which look alike are nonatomic, (74b) must always be false. This means that (74b) is contradictory—hence illformed on semantic grounds.\(^\text{32}\)

\(^{32}\) The nonsynonymy between the two \textit{N} and \textit{both N} is presumably borne out as well by the contrast \textit{one of the two books / ?one of both books} noticed in Ladusaw (1982), and by the contrast \textit{Jonathan found a poem stuck between the two pages of logical formulae / ?Jonathan found a poem stuck between both pages of logical formulae} discussed in Roberts (1986, 199).
The Semantics of Countability II

1 Introduction

One issue left outstanding in the preceding chapter was that of the 'denotation failure' of countable nouns. As the reader will recall, the proposals advanced in Chapter 3 led us to the conclusion that no countable stem could ever fail to denote (in an atomistic universe). But such a conclusion is most questionable; countable nouns should not denote just by virtue of being countable.

As it turns out, the proposals advanced in the preceding chapter in fact present us with a further problem. As is well known, languages may quantify over proper kinds, i.e. over individuals which have proper instances. Consider for instance the sentences in (1). As Heyer (1985, 67) has noted, they involve what medieval logicians called *distributio pro generibus singulorum*

(1) a. All animals were saved by Noah in his Ark.
   b. There are but four camelids in America: the llama, the alpaca, the vicuña, and the guanaco.

Notice that the sentence in (1a) may be true even if most 'actual animals' were not saved by Noah in his Ark. Similarly, the sentence in (1b) may be true even though 'actual camelids' in America number in the millions. The sentences in (1) are thus true only if they involve quantification over proper kinds—the proper kinds of animals in (1a) and the proper kinds of American camelids in (1b).

To account for the quantification exhibited by these sentences we will need to substantially generalize the proposals made in the preceding chapter. The purpose of the present chapter is to provide such generalizations. It will turn out that these provisions will solve the problem of the denotation failure of countable nouns, as they will allow the empty set as a possible countable stem denotation. Further evidence for the proposals to be advanced will be found in the next chapter, where we address the semantics of uncountability.
2 Countable Stems

Of all the subsets of a mereology, there will always be some which are closed under both sum and complementation.¹ Let us say now that such subsets are the submereologies of the mereology. To illustrate the notion of submereology we have just introduced, we will once again appeal to an atomistic universe whose mereological structure can be diagramed as follows.

(2)

\[
\begin{array}{c}
\text{a+b+c} \\
\text{a+b} & \text{a+c} & \text{b+c} \\
\text{a} & \text{b} & \text{c}
\end{array}
\]

Now, as defined in Chapter 2, a universe is a mereology. We may therefore speak coherently of the submereologies of a universe. We may speak, in particular, about the six submereologies of the universe diagramed in (2):

(3) \{\}, \{a+b+c\}, \{a, b+c, a+b+c\}, \{b, a+c, a+b+c\}, \{c, a+b, a+b+c\}, \{a, b, c, a+b, a+c, b+c, a+b+c\}.

Notice that both the empty subset and the full subset of a mereology are submereologies. Notice also that the set constituted solely by the universal kind (if there is one) is a submereology, and that this is the only possible singleton submereology. We will henceforth say that these submereologies are the empty submereology, the full submereology, and the universal submereology, respectively.

But let us return now to the domains in a universe. It will be recalled that a domain in a universe \(E\) is a set \(E \mid k = \{x \in E : x \leq k\}\), where \(k \in E\). It will also be recalled that every domain in a universe constitutes a mereology in its own right. Every domain in a universe will therefore have its own submereologies. Consider for instance (4),

---
¹ Let \(E\) be the field of a mereology and let \(F\) be a subset of \(E\). As one might expect, \(F\) is closed under sum just in case the following condition obtains for all \(G \subseteq F\). If \(\Sigma(G)\) is the sum of all elements of \(G\) (as taken in \(E\)), then \(\Sigma(G) \in F\). In addition, \(F\) is closed under complementation just in case all \(x \in F\) satisfy the following condition. If \(x'\) is the complement of \(x\) (as taken in \(E\)), then \(x' \in F\).
which tabulates the submereologies of each domain in our favorite universe.

<table>
<thead>
<tr>
<th>Domains</th>
<th>Submereologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E</td>
<td>a$</td>
</tr>
<tr>
<td>$E</td>
<td>b$</td>
</tr>
<tr>
<td>$E</td>
<td>c$</td>
</tr>
<tr>
<td>$E</td>
<td>a+b$</td>
</tr>
<tr>
<td>$E</td>
<td>a+c$</td>
</tr>
<tr>
<td>$E</td>
<td>b+c$</td>
</tr>
<tr>
<td>$E</td>
<td>a+b+c$</td>
</tr>
</tbody>
</table>

The first three domains in (4) have two submereologies each. They are the empty and the full submereologies (the full and the universal submereologies coincide for this domain). The next three domains in (4) have three distinct submereologies each. They are the empty, the full, and the universal submereologies. The last domain in (4) is $E$ itself. Consequently, it has the six submereologies which we have listed under (3) above.

Let us say now that the subdomains of an individual are nothing but the submereologies of the domain of that individual. Thus, the atom $a$ has two subdomains in our favored universe; they are { } and $\{a\}$. The proper kind $a+b$ has three subdomains in (2); they are { }, $\{a+b\}$, and $\{a, b, a+b\}$. The universal kind $a+b+c$ has six subdomains in (2); they are { }, $\{a+b+c\}$, $\{a, b+c, a+b+c\}$, $\{b, a+c, a+b+c\}$, $\{c, a+b, a+b+c\}$, and the universe itself.

Now, it should be noticed that every subdomain in a universe will constitute a mereology in its own right when taken in conjunction with the relation of instantiation of the universe. But such a mereology may of course be atomistic. We may therefore speak coherently of atomistic subdomains. We are finally prepared for the two conditions on nominal stems we wish to propose in this chapter. Consider then

(5) **Countable Stems**

Every countable stem denotes an atomistic subdomain in the universe.

Thus, any model which provides the universe in (2) and abides by (5) will provide the submereologies in (4) as possible countable stem denotations. A complete (and irredundant) listing of these denotations is given in (6).
(6) \{ \}, \{a\}, \{b\}, \{c\}, \{a+b\}, \{a+c\}, \{b+c\}, \{a+b+c\}, \{a, b, a+b\}, \{a, c, a+c\}, \{b, c, b+c\}, \{a, b+c, a+b+c\}, \{b, a+c, a+b+c\}, \{c, a+b, a+b+c\}, \{a, b, c, a+b, a+c, b+c, a+b+c\}

As a glance at the list of domains of (2) will reveal, (5) represents a substantial liberalization of the constraint on countable stems proposed in the preceding chapter. Yet, (5) still represents a strong constraint on the denotation of a countable stem. Of the one hundred twenty eight possible subsets of our universe, only fifteen may serve as countable stem denotations.

It should not escape the reader that (5) now allows for the empty set as a potential countable stem denotation. Hence the problem pertaining to 'denotation failure' in atomistic universes simply vanishes under the present proposals. We will now see that these proposals are in addition motivated independently of denotation failure. For, consider now the second condition on nominal stems we wish to propose.

(7) Nominal Stems

Let \(E|k\) be the domain of some individual \(k\) in some universe \(E\). Let \(P\) be a nonempty subdomain of \(k\) in \(E\). If there is a nominal stem which denotes \(E|k\), then there is another nominal stem which is homophonous to the first but denotes \(P\).

To illustrate the force of this constraint, let us consider any admissible model which provides the universe in (2) and interprets a nominal stem, call it ring\(_1\), as indicated in (8a). Let us consider, that is, a model which interprets a stem ring\(_1\) as \(E\), the entire universe in question. But \(E\) is also the domain of the universal kind \(a+b+c\). And, as we have seen, this individual has five nonempty subdomains. Hence (7) requires the model to provide the interpretations in (8b)–(8e) as well. The constraint in (7) thus requires ring to be five ways ambiguous.

(8) a. \([\text{ring}_1]\) = \(\{a, b, c, a+b, a+c, b+c, a+b+c\}\)

b. \([\text{ring}_2]\) = \(\{a, b+c, a+b+c\}\)

c. \([\text{ring}_3]\) = \(\{b, a+c, a+b+c\}\)

d. \([\text{ring}_4]\) = \(\{c, a+b, a+b+c\}\)

e. \([\text{ring}_5]\) = \(\{a+b+c\}\)

It will be noticed that the condition in (7) adds no denotations which have not been already contemplated in (6). Yet, (7) is far from otiose. First, it simplifies the presentations of models. For now models need stipulate only what might be called the 'primary meanings' of nominal stems; (7) can then be relied on for the generation of the 'secondary meanings' of these stems. More importantly, however, the condition in
(7) expresses an important generalization concerning the semantics of nominal stems. As will be seen below, this generalization will be key to our account of the quantification over kinds displayed in (1) above. In addition, it will play a central role in our account of definite generics—and further benefits of this generalization will accrue when we discuss the reinterpretation of uncountables as countables in Chapter 5.

Incidentally, if $k$ is a kind which is constituted by $n$ atoms of the universe, then it can be shown that $k$ will have $B(n)$ nonempty subdomains, where $B(n)$ is the $n$th Bell number—the versatile integer which counts the number of partitions of a set with $n$ elements, the number of factorizations of a number with $n$ distinct prime factors, and the number of possible rhyme schemes for a stanza with $n$ verses (cf. Gardner 1978). The condition in (7) thus generates $B(n) - 1$ 'secondary stems' for each 'primary countable stem'. Since Bell numbers grow exponentially, the condition in (7) constrains the exponential growth of the lexicon. Since the stems generated are all homophonous, the condition in (7) furthermore provides a principle for the exponential growth of ambiguity. See Appendix B.

3 Number Inflection

The generalization (5) of the constraint on countable stems presented in the preceding chapter calls for a corresponding generalization in the denotations of number inflections. This we provide in (9).

(9) **Number Inflections**

a. The singular inflection denotes a function which selects the atomic elements of each atomistic subdomain in the universe.

b. The plural inflection denotes a function which selects the atomistic elements of each atomistic subdomain in the universe.

As can be readily seen, the singular inflection selects the atoms of an atomistic subdomain, regardless of whether the selected elements are atoms of the universe. As to the plural inflection, it denotes a function which assigns each countable stem denotation to itself; it is the identity function over countable stem denotations.

As before, the semantic effect of number inflections will be best appreciated after we consider their interaction with the denotation of countable stems. In any event, the interpretations in (9) extend straightforwardly to account for dual, trial, and quadral inflections. Consider for instance the universal interpretations in (10).
(10) **Number Inflection (Generalized)**

a. The singular inflection denotes a function which selects the atomic elements of each atomistic subdomain in the universe.

b. The dual inflection denotes a function which selects the diatomic elements of each atomistic subdomain in the universe.

c. The trial inflection denotes the function which selects the tri-atomic elements of each atomistic subdomain in the universe.

d. The quadral inflection denotes the function which selects the tetratomic elements of each atomistic subdomain in the universe.

e. The plural inflection denotes a function which selects the atomistic elements of each atomistic subdomain in the universe.

4 Countable Nouns

Since countable stems denote subsets of the universe, and since number inflections now denote functions over such sets, the interpretation of a countable noun will now be governed by the Functional Principle, repeated here as (11).

(11) **The Functional Principle**

Whenever an expression which denotes a function combines with an expression which denotes a potential argument for that function, the denotation of the resulting expression is the actual application of the function to the potential argument.

To illustrate we consider (12), which contains the interpretations of two countable nouns relative to a model which abides by conditions (5), (9), (11). We assume that these nouns involve a countable stem *ring* which denotes an atomistic subdomain whose atoms are collected in a set $R$. In addition, we continue assuming the Asterisk Convention introduced in the preceding chapter.

(12)  

a. $\llbracket \text{ring} \rrbracket = \llbracket \emptyset \rrbracket (\llbracket \text{ring} \rrbracket ) = R$

b. $\llbracket \text{rings} \rrbracket = \llbracket s \rrbracket (\llbracket \text{ring} \rrbracket ) = R^*$

Let us say now that the interpretations in (12) have proceeded relative to a model which provides a universe which can be diagramed as indicated in (2) above. In addition, let us say that this model interprets a stem *ring* as the atomistic subdomain represented in Figure 1 below. Relative to this model, the singular noun *ring* will denote the set in Figure 2 below, whereas the plural noun *rings* will again denote the set in Figure 1.
But suppose now that this model in addition abides by (7), the general condition on nominal stems. It follows that there will be another stem *ring* which the model interprets as the atomistic domain indicated in Figure 3 (this stem was called *ring*₃ in (8) above). Under this interpretation, the singular noun *ring* would denote the set of atoms of this subdomain, namely the set represented in Figure 4; the plural *rings*, on the other hand, would denote the set in Figure 3 itself.

To make all this more concrete, let us say that the universe contains but two gold rings and one silver ring. Let us moreover say that the gold rings are \( a \) and \( c \) whereas the silver ring is \( b \). Under one of its readings, the singular noun *ring* denotes \( \{a, b, c\} \), the set constituted by all the atomic rings of the universe. Under another reading, however, the singular noun *ring* will denote \( \{a+c, b\} \), the set constituted by the (proper) kind of gold rings and the (improper) kind of silver rings.

To further illustrate the effect of the constraints advanced in this chapter, we will turn again to Classical Greek. Consider for instance (13), which contains the interpretations of the singular noun *daktulios*...
Linguistic Individuals

‘ring’, the dual daktyliō ‘two rings’, and the plural daktulioi ‘rings’. Here the interpretation proceeds relative to a model which satisfies conditions (5), (10), (11), provides a set $R$ of rings, and provides a set $2R$ of binary sums of rings.

\[(13)\quad \begin{align*}
a. & \, \llbracket daktulios \rrbracket = \llbracket os \rrbracket (\llbracket daktuli \rrbracket) = R \\
b. & \, \llbracket daktyliō \rrbracket = \llbracket ō \rrbracket (\llbracket daktyli \rrbracket) = 2R \\
c. & \, \llbracket daktulioi \rrbracket = \llbracket oi \rrbracket (\llbracket daktuli \rrbracket) = R^* \\
\end{align*}\]

Let us suppose now that the interpretation in (13) proceeded relative to a model which provided the universe in (2) and which interpreted the stem daktuli as the atomistic subdomain in Figure 3 above. It follows that this set is the denotation of the plural noun daktulioi, whereas the set in Figure 4 is the denotation of the singular daktulios. As to the dual daktyliō it denotes the singleton set represented in Figure 5 below.

5 Singularity as Disjointness

It will be recalled that two kinds were said to be disjoint just in case they shared no common instances. Let us say now that a subset of the universe is pairwise disjoint if and only if any two distinct elements thereof are disjoint. If the singular noun ring denotes the set in Figure 2, it denotes a set of atomic elements; if the noun ring denotes the set in Figure 4 it denotes only a set of pairwise disjoint elements—it should be clear that every set of atomic elements is pairwise disjoint but not every set of pairwise disjoint elements is atomic.

Cast in more general terms, every singular noun will denote a pairwise disjoint subset of the universe.\(^2\) Conversely, every pairwise disjoint subset of the universe is denoted by a singular noun.

\(^2\)Here is a proof. Let $F$ be the denotation of a singular noun and let $x$ and $y$ be two distinct elements of $F$. Suppose that $x$ and $y$ overlap. By the mereological equivalent of the De Morgan laws (cf. Leonard and Goodman 1940, 50), the
disjoint subset of the universe is a potential singular noun denotation.\(^3\)

It follows that singularity is pairwise disjointness or, more precisely, that a noun is singular if and only if it denotes a pairwise disjoint subset of the universe. From this it follows that the number of nonvacuous singular nouns in an atomistic universe is again given by the Bell numbers. To be more precise, it can be shown that any model which provides an atomistic universe with \(n\) atoms can interpret exactly \(B(n+1)\) distinct singular, nonvacuous, nouns, where \(B(n+1)\) is the \(n+1\)th Bell number. See again Appendix B.

Our discussion of the semantics of countability began by acknowledging, after Jespersen (1924), the existence of expressions which ‘called up the idea of some definite thing with a certain shape or precise limits’. As we see it, these ‘things with precise limits’ should be construed as the elements of pairwise disjoint subsets of the universe. Some of these pairwise disjoint sets will be constituted by atoms of the universe—but not all will.

6 Markedness and Plurality

Let us say that \([\text{SINGULAR}]\) is the denotation of some singular noun while \([\text{PLURAL}]\) is the denotation of its plural counterpart. We observe that the inclusion in (14) will hold in any model which abides by conditions (5), (9), (11).

\[(14) \quad [\text{SINGULAR}] \subseteq [\text{PLURAL}]\]

It should be clear that (14) is identical to the inclusion given under (12) in Chapter 3. Yet, this identity pertains only to form, as the sets overlap or product \(xy\) of \(x\) and \(y\) is equal to \((x'+y')'\). Since submereologies are by definition closed under sum and complementation, \(xy\) must therefore belong to any submereology to which \(F\) belongs. Since \(x\) and \(y\) are distinct, then \(xy < x\) and \(xy < y\). But then \(F\) cannot be a set of atoms (i.e. minima) of any submereology. Hence \(F\) cannot be the denotation of a singular noun.

\(^3\) Here is a proof (the reader should beware that the denotation of singular nouns will be further constrained in Chapter 6). Let \(P\) be a pairwise disjoint subset of the universe. It will be the set of atoms of \(P^*\). \(P\) will therefore be a singular noun denotation if \(P^*\) is a subdomain in the universe. This is immediate if \(P^*\) is empty (\(P^*\) is the empty subdomain of any individual). So suppose \(P^*\) is not empty. We will show that \(P^*\) is a subdomain of \(\Sigma(P)\). Take first any \(Q \subseteq P^*\). Consider the elements of \(P\) which instantiate some element of \(Q\). The sum of these elements is in \(P^*\) (by definition of \(P^*\)). So \(\Sigma(Q) \in P^*\), and \(P^*\) is closed under sums as taken in \(E \mid \Sigma(P)\). But take now any \(x \in P^*\) other than \(\Sigma(P)\); \(x\) will have a complement \(x'\) in \(E \mid \Sigma(P)\). Let \(Q\) be the set of elements of \(P\) which instantiate \(x\). Clearly, \(x' = \Sigma(P - Q)\). But \(\Sigma(P - Q) \in P^*\) (by definition of \(P^*\)). So \(x' \in P^*\), and \(P^*\) is closed under complementation as taken in \(E \mid \Sigma(P)\).
mentioned here are clearly different from the ones mentioned before. Still, the converse of (14) will not hold in all models which abide by conditions (5), (9), (11). We can therefore conclude now as we did then, that the plural is the semantically unmarked member of the opposition of number in every model which abides by these conditions. It follows that all the data pertaining the indeterminacy of the plural with respect to number can still be accounted for. It also follows, however, that all the issues left open by the previous theory of plurality will remain open under the present setting.

We conclude this section by considering an alternative theory of plurality—one which is the transposition of the theory of Link (1983, 306) into a mereological setting. The main attraction of this theory is that it avoids the puzzling mismatch between markedness of form and markedness of meaning inhering in the plural.

The point of departure of the theory of plurality found in Link (1983) is the assumption that every countable stem denotes a set of atoms of the universe (or a set of pairwise disjoint elements of the universe, if we were to follow the line of argumentation presented in this chapter). Starting from this assumption, we can then “take seriously the morphological change in pluralization, which is present in many natural languages, and introduce an operator, "∗", working on 1-place predicates $P$, which generates all the individual sums of members of the extensions of $P$”. Plural nouns are thus interpreted as shown in (15), where $R$ is again an atomic (or else pairwise disjoint) set of rings.

\[ [\text{rings}] = [s] ([\text{ring}]) = [\text{PLURAL}] (R) = R^* \]

On the other hand, the singular inflection, which is phonetically void in the unmarked case, will be semantically void in the general case: it will be interpreted as the identity function over countable stem denotations. Singular nouns are thus interpreted as illustrated in (16).

\[ [\text{ring}] = [\emptyset] ([\text{ring}]) = [\text{SINGULAR}] (R) = R \]

As might be expected, then, the unmarked meaning (the identity function) takes up the unmarked form, whereas the marked meaning (the completion function) takes up the marked form.

But atomicity (or pairwise disjointness) is not always attained through phonetically void means. Consider for instance languages as diverse as Kwakiutl, Japanese and Warrgamay. Here nouns are not generally inflected for number; yet their denotations are not semantically singular, but rather neutral with respect to the semantic distinction between singular and plural (cf. Boas 1911, 33; Martin 1975, 143;
Dixon 1980, 267). Notice that the facts of these languages are not those of a reversal of number marking, where singulars are marked overtly whereas plurals are not. Rather, the facts are those of the absence of number marking. And when number inflection is absent, 'plurality' ensues. Needless to say, these facts are not only compatible with the proposals of this study; they are predicted by them.

But it might be thought that the semantics of number calls for two kinds of countable stems. Languages like English would call for 'individual' stems of the kind envisaged by Link, whereas languages like Kwakiutl, Japanese, and Warrgamay would call for 'collective' stems of the sort proposed in this study. Needless to say, such solutions are overstipulative by comparison to the proposals advanced in this work. In addition, such solutions will hinder the formulation of the general constraints on the semantics of nominal stems to be proposed in Chapter 6.

7 Cumulative Reference

The proposals presented in this chapter do not affect the definition of cumulative reference presented in Chapter 3. As to the cumulative reference of plural nouns, it can still be accounted for: atomistic subdomains are mereologies in their own right (just as atomistic domains were); they are therefore complete. As to singular nouns, they will again denote cumulatively just in the trivial case that they denote a singleton (only then will a pairwise disjoint set, atomic or otherwise, be complete).

8 Semantic Parsimony

The proposals presented in this chapter preserve the advantages which a first order theory of plurality offers over the second order theory of Bennett (1974): it is semantically parsimonious without having to resort to an unmotivated increase in abstractness.

9 Cardinal Adjectives

Let us turn now to universal interpretations for cardinal adjectives in the present, generalized, setting. To do so we need to relativize our function \( \mu \) to subdomains of the universe (thereby generalizing its applicability). Suppose then that \( P \) is an atomistic subdomain in a
universe \( E \). We will say that \( \mu(P) \) is a function which assigns, to each \( x \in P \), the number of atoms of \( P \) which constitute \( x \).

(17) **Cardinal Adjectives**

The cardinal adjective \( n \) denotes a function which assigns, to each atomistic subdomain \( P \subseteq E \), the set \( \{ x \in P : [\mu(P)](x) = n \} \),

\[ \text{where } n \text{ is the } n\text{th positive integer.} \]

Every cardinal adjective \( n \) thus denotes a function which selects those elements of a set \( P \) (if there are any) which are constituted by \( n \) atoms of \( P \). It should be emphasized that \( [\mu(P)](x) \) is the number of atoms of \( P \) in \( x \); the atoms of \( P \) may or may not be atoms of the universe.

To illustrate the effect of (17), we will point out that any model which abides by conditions (5), (9), (11), (17) will induce the interpretations in (18) below, where \( R \) is the denotation of a singular noun ring.

(18) a. \( \left[ \text{one ring} \right] = \left[ \text{one} \right] \left( \left[ \text{ring} \right] \right) = \left[ \text{one} \right] (R^*) = R \)

b. \( \left[ \text{two rings} \right] = \left[ \text{two} \right] \left( \left[ \text{rings} \right] \right) = \left[ \text{two} \right] (R^*) = 2R \)

c. \( \left[ \text{three rings} \right] = \left[ \text{three} \right] \left( \left[ \text{rings} \right] \right) = \left[ \text{three} \right] (R^*) = 3R \)

d. \( \left[ \text{four rings} \right] = \left[ \text{four} \right] \left( \left[ \text{rings} \right] \right) = \left[ \text{four} \right] (R^*) = 4R \)

Now, it might seem that \( R^* \) in (18a) proceeds from a plural noun which is nowhere to be found. But following Krifka (1987, 10), we will claim that the number inflection on nouns modified by cardinal adjectives is a semantically empty inflection demanded by the numerals. This means that \( R^* \) in (18a) proceeds from an uninflected stem ring, not from a plural noun rings.

To argue for his claim, Krifka points to phrases like zero cows and one point zero cows, "which have nothing to do with the semantic concept of plurality but are easily explained if one assumes that zero and one point zero trigger syntactic plurality". To this we might add that *one scissors is ungrammatical even when we refer to a single artifact, and that the principles for assigning number to nouns in construction with numerals vary significantly across the languages of the world—in the Arabic spoken in Iraq, for example, "immediately following any numeral from three through ten the noun follows and is plural," but "immediately following any numeral over ten, the noun follows and is singular" (cf. Erwin 1963, 263).

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4 If \( P \) is an atomistic domain and \( x \in P \), then \( \mu \) is equivalent to a two-place function which assigns, to each pair \( <P, x> \), the number of atoms of \( P \) which constitute \( x \)

5 Further evidence for the claim is provided by the semantics of uncountable and classifier constructions. See Chapter 5, Section 12, and Chapter 6, Note 8.
Let us say now that the interpretations in (18) have proceeded relative to a model which provides the universe diagramed in (2). Let us moreover assume that this model interprets a stem \textit{ring} as the subset represented in Figure 1 above. Relative to this model, (18a) would be the set in Figure 2 above; (18b) would be the set in Figure 6 below; (18c) would be the set in Figure 5 above, and (18d) would be the empty set, as there are no four rings in this model.

![Figure 6](image.png)

But suppose now that this model in addition abides by (7), the general condition on nominal stems. It again follows that there will be another stem \textit{ring} which the model interprets as the set indicated in Figure 3 above. Relative to this model, (18a) would be the set in Figure 4 above; (18b) would be the set in Figure 5 above; (18c) would be the empty set, as there are no three rings \textit{in this sense} in the universe. \textit{A fortiori}, (18d) would also be the empty set.\[6\]

Equipped with the foregoing we may now address the sentences in (1)—particularly the sentence in (1b). Let us consider a model which provides twenty million American camelids. These camelids would be the atoms of the domain of the kind constituted by all American camelids. This domain would be vast; it would contain $2^{20,000,000} - 1$ elements. Let us pick out four elements of this forbidding domain. They are the kind of llamas, the kind of alpacas, the kind of vicuñas, and the kind of guanacos. Let us suppose now that these four kinds are what they in fact are: pairwise disjoint and constitutive of the entire kind of American camelids. It can be shown that these four kinds are the atoms of a \textit{subdomain} of the kind constituted by all American camelids. This

\[6\] Everything that has been said about \textit{one ring}, \textit{two rings}, \textit{three rings} and \textit{four rings} can be said about \textit{one thing}, \textit{two things}, \textit{three things} and \textit{four things}, as \textit{ring} and \textit{thing} are synonymous in this model.
subdomain will be immensely more manageable, as it would have only $2^4 - 1 = 15$ elements. Its structure can be diagramed as follows.

(19)

If this model abides by (7), then the stem *American camelid* would denote the subdomain diagramed in (19).\footnote{We indulge here in an obviously false simplification, as *American camelid* is not a stem. Readers wanting to eliminate this simplification are invited to replace *American camelid* by *auchenid* throughout.} Moreover, if the model abides by (17) and other independently motivated conditions, we would have the following denotations:

(20) a. \[[one American camelid]] = \{a, b, c, d\} 

b. \[[two American camelids]] = \{a+b, a+c, a+d, b+c, b+d, c+d\} 

c. \[[three American camelids]] = \{a+b+c, a+b+d, a+c+d, b+c+d\} 

d. \[[four American camelids]] = \{a+b+c+d\} 

If *American camelid* is interpreted as the set in (19), then it is indeed true that there are but four camelids in America; if, however, *American camelid* is interpreted as the entire domain of the kind of American camelids (the one with $2^{20,000,000} - 1$ elements) then (1b) is blatantly false.

It might be objected that there is nothing in our account of *there are but four camelids in America* which explains how the subdomain diagramed in (19) arose from the myriad alternative subdomains. But this is as it should be. What explains the rise of (19) is the sequel of this sentence, namely, *the llama, the alpaca, the vicuña, and the*
guanaco; when taken in isolation, the first part of (1b) is hopelessly ambiguous against a model which provides for millions of American camelids. Only when taken in context does the ambiguity disappear.\footnote{And there may well be other factors which contribute to the process of disambiguation in actual performance. Thus, a case can probably be made that the probability of selecting a particular sense increases dramatically if the subkinds on which it is based are lexicalized, so the existence of the nouns llama, alpaca, vicuna, and guanaco would assign the subdomain diagramed in (19) a 'selectional probability' which is much greater than average (I am indebted to Greg Carlson for this observation). It is to be furthermore expected that the principles guiding the erection of 'folk taxonomies' will also contribute to the selectional probability of particular senses in the present context.}

But should the massiveness of this ambiguity, then, be cause for concern? It might be thought that by proposing a vast (and possibly infinite) number of readings for nouns will require a correspondingly vast (and possibly infinite) number of lexical stipulations (with the subsequent rejection of grammars as finitary formalisms). But this would obtain only if each lexical reading would have to be independently represented by the grammar. Nothing of the sort is called for by our account of quantification over kinds, which proposes a single, very general, principle for deriving secondary meanings from a finite number of primary meanings. Only the finitely many primary meanings would have to be independently represented by the grammar.

Or, it might be objected that any sentence of the form there are but \(n\) camelids in America will be true in models with at least \(n\) camelids in America (if there are at least \(n\) camelids, then they can always be classified into \(n\) kinds). But notice that each of these sentences will involve a different sense of camelid in America. So the sentences in question are indeed devoid of empirical consequence unless and until the relevant domain of quantification, the sense of camelid in America, is fixed. But this should be unobjectionable: how many there are depends on what you count.

We conclude this section with a number of observations. Notice first that the present, generalized, setting still allows \([\text{one}]\), \([\text{two}]\), \([\text{three}]\), and \([\text{four}]\) to be respectively equal to \([\text{SINGULAR}]\), \([\text{DUAL}]\), \([\text{TRIAL}]\), and \([\text{QUADRAL}]\), the generalized denotations of number inflections given now in (10) above.

Second, notice that the universe \(E\) is itself a subdomain. Hence a function \(\mu(E)\) is available; it may coincide with \(\mu\) as defined in the preceding chapter. Third, note that we shall distinguish the semantics of cardinal adjectives from the semantics of cardinal nouns which are...
homophonic thereto.\textsuperscript{9} As we have seen, cardinal adjectives are interpreted as indicated in (17); cardinal nouns, on the other hand, may be interpreted as cardinal adjectives were in effect interpreted in the preceding chapter:

(21) *Cardinal Nouns*

The cardinal noun $n$ denotes the set \( \{x \in E : [\mu(E)](x) = \nu \} \), where $\nu$ is the $n$th positive integer.

The cardinal noun $n$ will now be predictably synonymous with the nominal phrase $n$ thing(s).\textsuperscript{10}

\section*{10 Multal and Paucal Adjectives}

To work adequately in the present setting, the interpretation of multal and paucal adjectives presented in the preceding chapter must be generalized to arbitrary subdomains in the universe. This is what we do in (22) below.

(22) *Multal and Paucal Adjectives (Countable Version)*

(a) The adjective *many* denotes the function which assigns, to each atomistic subdomain $P$, the set \( \{x \in P : [\mu(P)](x) \text{ is large in } P \} \).

(b) The adjective *few* denotes the function which assigns, to each atomistic subdomain $P$, the set \( \{x \in P : [\mu(P)](x) \text{ is small in } P \} \).

Thus, when presented with an appropriate set $P$, the multal adjective *many* selects those elements of $P$ which are sums of a large number of atoms of $P$. The paucal adjective *few*, on the other hand, selects those elements of $P$ which are sums of a small number of atoms of $P$.

It should be clear that $[\mu(P)](x)$ will be large or small only with respect to $P$. Thus, it would seem natural to say that $x$ would be in the denotation of *many American camelids* if $[\mu(P)](x) = 3$ and $P$ is the subdomain in (19). It would seem just as natural to say, however, that $x$ would not be in the denotation of the homophonous phrase if $[\mu(P)](x) = 3$ and $P$ were instead the full domain of the kind of American camelids of which (19) is a subdomain. It should also be clear that all the points made in the preceding chapter concerning contextualization,

\textsuperscript{9} Distributionally, nouns may occur in absolute position (cf. *one is an even number* or *two is less than three*), whereas cardinal adjectives may not.

\textsuperscript{10} See Chapter 5, Section 16, for further discussion of constructions with cardinal adjectives.
fuzziness, and ambiguity of multal and paucal adjectives carry over to the present, more general, setting.\textsuperscript{11}

\section*{11 Some Assumptions}

We will continue to assume with Montague (1973) that every (meaningful) verb phrase denotes a subset of the universe, that every (meaningful) noun phrase denotes a set of subsets of the universe, and that every (meaningful) sentence asserts that the denotation of its predicate verb phrase is an element of the denotation of its subject noun phrase. We will thus continue to assume the conditions repeated here as (23) and (24).

(23) a. Let $VP$ be a verb phrase. If $\llbracket VP \rrbracket$ is defined, then it is a subset of $E$.

b. Let $NP$ be a verb phrase. If $\llbracket NP \rrbracket$ is defined, then it is a set of subsets of $E$.

(24) Let $S$ be a sentence constituted by an $NP$ and a $VP$. If $\llbracket NP \rrbracket$ and $\llbracket VP \rrbracket$ are defined, $S$ is true if $\llbracket VP \rrbracket \in \llbracket NP \rrbracket$ and false if $\llbracket VP \rrbracket \notin \llbracket NP \rrbracket$.

\section*{12 Universal Determiners}

Let us turn now to the universal determiners, which we should now interpret as shown in (25). As usual, we proceed here relative to a universe $E$.

(25) \textbf{Universal Determiners}

a. The determiner \textit{every} denotes a function which assigns, to each pairwise disjoint $P \subseteq E$, the family $\{X \subseteq E : P \subseteq X\}$.

b. The determiner \textit{all} denotes a function which assigns, to each $P \subseteq E$, the family $\{X \subseteq E : P \subseteq X\}$.

As before, universal determiners are similar in that they construct the supersets (relative to the universe) of a given set. They are different, however, as to the sets they can construct the supersets of. \textit{Every} constructs the supersets of pairwise disjoint subsets of the universe while \textit{all} constructs the supersets of arbitrary sets of individuals. It will also be noticed that (25) is nothing but a generalization of the previous condition on universal determiners. Whereas the previous condition restricted \textit{every} to atomic subsets, the present condition restricts it to

\textsuperscript{11} See Chapter 5, Section 16, for further discussion of constructions with multal and paucal adjectives.
pairwise disjoint subsets. Cast in terms of domains and subdomains, the previous condition restricted every to the sets of atoms of the domains in the universe; the present condition restricts it to the sets of atoms of the subdomains in the universe.

To illustrate the external semantics of universally quantified noun phrases, let us consider now the sentence in (26).

(26) Every ring is expensive.

Let us moreover consider a model whose universe is as diagramed in (2) and which interprets the noun ring as the set \( \{b, a+c\} \) enclosed in Figure 4 above. Let us finally suppose that the set of entities which are expensive in this universe is \( \{b, c, b+c\} \), which is the set that has been represented in Figure 7:

![Figure 7](image)

It follows from (24) and (25a) that sentence (26) will be true if \( \{b, c, b+c\} \) is a superset of \( \{b, a+c\} \) and false if it is not. But \( \{b, c, b+c\} \) is clearly not a superset of \( \{b, a+c\} \). Hence sentence (26) is false relative to the model in question. Moreover, if \( \text{is expensive} \) is not a superset of \( \text{ring} \) in this model, then it will certainly not be a superset of \( \text{rings} \), the more inclusive set represented in Figure 3 above. And more generally, whenever rings is taken in its general sense, then the falsity of (26) will entail the falsity of (27).

(27) All rings are expensive.

To further illustrate the external semantics of universal determiners we will turn now to the sentence in (28a). It will be recalled that we proposed that look alike denotes a function which assigns, to each domain in the universe, the set of nonatoms of that domain which look alike. We should now generalize this proposal and say that look alike denotes a function which assigns, to each subdomain in the universe, the set of nonatoms of that subdomain which look alike. Given (24),
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(25), and the semantics of look alike just proposed, the truth conditional contribution of (28a) will be as shown in (28b).

(28) a. All rings look alike.
   b. \[[rings] \subseteq [\text{look alike}] ([\text{ring}])\]

Let us say now that the universe is as diagramed in (2). Let us moreover say that the plural on rings was taken in its specific meaning,\(^2\) so that rings can denote the set \(\{a+b+c\}\) of Figure 5. Let us say, finally, that the set of rings which look alike in this universe is (again) the set \(\{a+b+c\}\) of Figure 5. It follows that the set of rings which look alike is an (improper) superset of the set of rings, and (28a) is therefore true.

But consider now the sentence in (29a). Given (24) and (25), its assertion would be as indicated in (29b)—at least if we continue to assume that the singular inflection on looks alike is a syntactic reflex of no semantic consequence.

(29) a. ?Every ring looks alike.
   b. \[[\text{ring}] \subseteq [\text{looks alike}] ([\text{ring}])\]

But then \([\text{looks alike}] ([\text{ring}])\) is a set of nonatoms of the subdomain denoted by the stem ring while \([\text{ring}]\) is the set of atoms of the same subdomain. This means that (29a) can only be true if there are no rings in the universe (for then the assertion is simply that \(\emptyset \subseteq \emptyset\)). (29a), then, is either false or else vacuously true. It is therefore ill formed.

Equipped with the foregoing, we can now return to the sentence in (1a). Consider the set of all animals which populated the Earth at the time of Noah. These animals constitute the atoms of a vast domain in the universe—the domain of the kind constituted by all animals. Suppose that such a domain is the denotation of the stem animal. Suppose also that the sentence in (1a) is interpreted relative to a model which abides by (7). It follows that the stem animal will be tremendously ambiguous, as it will have one sense for each of the subdomains of the kind of all animals. Now, of all these subdomains, there will have to be one whose atoms are the species which populated the Earth at the time of Noah (the set of animal species is pairwise disjoint and constitutive of the kind constituted by all animals). This subdomain is the set which provides the basis for the quantification in the most natural reading of (1a)—the one which renders it true.

\(^2\) See Chapter 3, Section 6, for a discussion of the general and the specific meanings of the plural.
It should be noticed that we can represent the fact that the animal couples entering Noah’s Ark are only instances of the species he saved; they are not the species he saved. In fact, the set of couples which entered the Ark is only a pairwise disjoint subset of the animal kingdom. It is therefore the set of atoms of some subdomain in the universe. Yet, it is not the set of atoms of a subdomain of the kind of all animals, as it is not constitutive of this kind.\(^{13}\)

We conclude this section with the observation that the inclusion in (30) will obtain for every pairwise disjoint subset \(P\) of the universe.

(30) \([\text{all}]\ (P^*) \subseteq [\text{every}]\ (P)\)

For, if some set is a superset of \(P^*\), then it will certainly be a superset of the less inclusive set \(P\) as well. Naturally, the converse of this inclusion will not obtain for all values of \(P\).

13 Existential Determiners

Let us turn now to the unstressed existential determiners \(a\) and \(some\), which we should interpret now as shown in (31). We once again proceed relative to a model which provides a universe \(E\).

(31) *Existential Determiners*

\[
\text{a. The determiner } a \text{ denotes the function which assigns, to each pairwise disjoint } P \subseteq E, \text{ the family } \{X \subseteq E: P \cap X \neq \emptyset\}. 
\]

\(^{13}\) It is interesting to compare this discussion with the *solutio* offered by Peter of Spain in his *Tractatus*, Chapter XI, Paragraph 14:

*Solution.* Some say 'every animal was in Noah’s Ark' is ambiguous [duplex] because there can be distribution for singulars within genera, or for genera of singulars [pro generibus singulorum]. In the first way it is false, since when distribution is for singulars of genera, distribution is then for all individuals under the genus and species. Then it would be appropriate for all individuals contained under ‘animal’ to have been in Noah’s Ark. And that is false. But when distribution is made for genera of singulars, there is only distribution for genera or species. But there was no species of animal which would not have been in Noah’s Ark. And in that way, the proposition is true.

I do not agree with that solution, because animal species were not of themselves in Noah’s Ark, only individuals. So it did not have truth at that time except for single members of genera, that is, for individuals, not for genera of singulars. So I say the [proposition] simply taken is false, and concede all arguments for this. [The argument that Man was in Noah’s Ark, Horse and Cow were in Noah’s Ark, and so on for each animal] offends by Fallacy of Consequent from Insufficient Enumeration, because it does not accept all parts of the distribution which are in the subject of the proposition ‘every animal was in Noah’s Ark’.
b. The determiner *some* denotes the function which assigns, to each $P \subseteq E$, the family $\{X \subseteq E : P \cap X \neq \emptyset\}$.

As can be readily seen, existential determiners are similar in that they construct the conjoints (relative to the universe) of a given set. They are different, however, as to the sets they can construct the conjoints of. The determiner *a* constructs the conjoints of pairwise disjoint subsets of the universe, while the determiner *some* constructs the conjoints of arbitrary sets of individuals. It will again be noticed that (31) is nothing but a generalization of the previous condition on existential determiners. Whereas the previous condition restricted *a* to atomic subsets, the present condition restricts it to pairwise disjoint subsets. Cast in terms of domains and subdomains, the previous condition restricted *a* to the sets of atoms of the domains in the universe; the present condition restricts it to the sets of atoms of the subdomains in the universe.

To illustrate the external semantics of existentially quantified noun phrases, we will consider now the sentence in (32).

(32) A ring is expensive.

Let us moreover consider a model whose universe is as diagramed in (2) and which interprets the noun *ring* as the set $\{b, a+c\}$ enclosed in Figure 4 above. Let us suppose, finally, that the set of entities which are expensive in this universe is $\{b, c, b+c\}$, the set which has been represented in Figure 7 above. It follows from (24) and (31a) that the sentence in (32) will be true if $\{b, c, b+c\}$ is a conjoint of $\{b, a+c\}$ and false if it is not. But $\{b, c, b+c\}$ clearly overlaps with $\{b, a+c\}$. Hence (32) is true relative to the model in question. Moreover, if $\langle \text{is expensive}\rangle$ is a conjoint of $\langle \text{ring}\rangle$ in this model, then it will certainly be a conjoint of $\langle \text{rings}\rangle$, the more inclusive set represented in Figure 3 above. And more generally, if *rings* is taken in its general sense, then the truth of (32) entails the truth of (33).

(33) Some rings are expensive.

To further illustrate the external semantics of existentially quantified noun phrases we will turn to the sentence in (34a).

(34) a. Some rings look alike.

b. $\langle \text{rings}\rangle \cap \langle \text{look alike}\rangle (\langle \text{ring-}\rangle) \neq \emptyset$

It will be noticed that the truth value of this sentence will depend on the model against which it is interpreted. For notice that (34a) will be true only in models in which the set of rings which look alike properly intersects the set denoted by *rings*. This is indicated in (34b). But suppose now that *rings* denotes the set $\{b, a+c, a+b+c\}$ enclosed in
Figure 3 above. Suppose further that the set of rings which look alike is the set \{a+b+c\} of Figure 5. Since these two sets overlap, (34a) would be true relative to this model. But consider a model in which the set of rings which look alike is empty. Relative to this model, the set of rings which look alike does not properly intersect the set denoted by rings. (34a) would thus be false relative to this model.

The truth of (34a) is thus dependent upon the models against which it is evaluated. But consider now the sentence in (35a), whose truth conditional contribution is as indicated in (35b).

(35) a. ?A ring looks alike.
   b. \[\{\text{ring}\} \cap \{\text{looks alike}\} (\{\text{ring}\}) \neq \emptyset\]

The truth value of this sentence does not depend on the model against which it is interpreted. To see this recall that \(\{\text{looks alike}\} (\{\text{ring}\})\) must always be a set of nonatoms of the subdomain denoted by the stem \text{ring}. Recall also that \(\{\text{ring}\}\) must always be the set of atoms of the same subdomain. This means that (35a) will be false in every model (even in those in which there are no rings). (35a) will thus be contradictory and therefore ill formed.

We wish to close this section with two observations. The first is that the inclusion in (36) will obtain for every pairwise disjoint subset \(P\) of the universe.

(36) \[\{a\} (P) \subseteq \{\text{some}\} (P^*)\]

For, if some set is a conjoint of \(P\), then it will certainly be a conjoint of the more inclusive set \(P^*\) as well. Naturally, the converse of this inclusion will not obtain for all values of \(P\).

Our second observation pertains the inclusions in (37), where \(P\) is a nonempty pairwise disjoint subset of the universe.

(37) a. \[\{\text{every}\} (P) \subseteq \{a\} (P)\]
   b. \[\{\text{all}\} (P^*) \subseteq \{\text{some}\} (P^*)\]

For every superset of a nonempty set is a conjoint thereof (though not conversely).

14 Nonexistential Determiners

The proposals presented in this chapter do not affect the interpretation of the nonexistential determiner presented in the preceding chapter. We will therefore repeat our proposal in (38) and show that it is compatible with the generalizations we have been making throughout this chapter.

(38) The Nonexistential Determiner

The determiner \textit{no} denotes the function which assigns, to each \(P \subseteq E\), the family \(\{X \subseteq E: P \cap X = \emptyset\}\).
Consider first the sentence in (39). Consider next a model whose universe is as diagramed in (2) and which interprets the noun ring as the set represented in Figure 4 above. Let us suppose, finally, that the set of entities which are expensive in this universe is the set which has been represented in Figure 7 above.

(39) No ring is expensive.

It follows from (24) and (38) that the sentence in (39) will be true if \( \{b, c, b+c\} \) is a disjoint of \( \{b, a+c\} \) and false if it is not. But \( \{b, c, b+c\} \) and \( \{b, a+c\} \) are clearly not disjoint. Hence (39) is false relative to the model in question. Moreover, if \( [[\text{is expensive}]] \) is not a disjoint of \( [[\text{ring}]] \) in this model, then it will certainly not be a disjoint of \( [[\text{rings}]] \), the more inclusive set represented in Figure 3 above. And more generally, whenever rings is taken in its general sense, then the falsity of (39) will entail the falsity of (40).

(40) No rings are expensive.

To further illustrate the external semantics of nonexistential determiners, we will turn to the sentence in (41a).

(41) a. No rings look alike.

\[ [[\text{rings}]] \cap [[\text{look alike}]](\bullet) = \emptyset \]

It will be noticed that the truth value of this sentence will depend on the model against which it is interpreted. For, given the semantics of look alike presented above, (41a) will be true only in models in which the set of rings which look alike is disjoint from the set denoted by rings. This is indicated in (41b). But suppose now that rings denotes the set \( \{b, a+c, a+b+c\} \) of Figure 3. Suppose further that the set of rings which look alike is empty. Since these two sets are disjoint, (41a) would be true in this model. But consider a model in which the set of rings which look alike would be the set \( \{a+b+c\} \) of Figure 5. Relative to this model, the set of rings which look alike is not disjoint from the set denoted by rings. (41a) would thus be false relative to this model.

The truth value of (41a) is thus dependent upon the models against which it is evaluated. But consider now the sentence in (42a), whose truth conditional contribution is as indicated in (42b).

(42) a. No ring looks alike.

\[ [[\text{ring}]] \cap [[\text{looks alike}]](\bullet) = \emptyset \]

The truth value of this sentence does not depend on the model against which it is interpreted. To see this recall that \( [[\text{looks alike}]](\bullet) \) must always be a set of nonatoms of the subdomain denoted by the stem ring. Recall also that \( [[\text{ring}]] \) must always be the set of atoms of
the same subdomain. This means that (42a) will be true in every model (even in those in which there are no rings). (42a) will thus be valid. The extreme uninformativeness of validity can then account for the peculiarity of (42a).\textsuperscript{14}

We conclude this section with the observations that the equalities in (43) and (44) will obtain for every pairwise disjoint subset \( P \) of the universe and for \( \emptyset(E) \) the set of all subsets of the universe.

\[(43)\]

\begin{align*}
\text{a. } & \llbracket a \rrbracket (P) \cap \llbracket \text{no} \rrbracket (P) = \emptyset \\
\text{b. } & \llbracket a \rrbracket (P) \cup \llbracket \text{no} \rrbracket (P) = \emptyset (E)
\end{align*}

\[(44)\]

\begin{align*}
\text{a. } & \llbracket \text{some} \rrbracket (P^*) \cap \llbracket \text{no} \rrbracket (P^*) = \emptyset \\
\text{b. } & \llbracket \text{some} \rrbracket (P^*) \cup \llbracket \text{no} \rrbracket (P^*) = \emptyset (E)
\end{align*}

(the set of conjoints of any set is mutually exclusive and jointly exhaustive with the set of disjoints of that set).

15 Dual Determiners

We may now turn to the dual determiners both and neither, which we wish to interpret as shown in (45). Once again, these interpretations proceed relative to a universe \( E \).

\[(45)\] Dual Determiners

\begin{align*}
\text{a. } & \text{The determiner } \text{both} \text{ denotes a function which assigns, to each } P^*, \text{ the family } \{X \subseteq E: P \subseteq X\}, \text{ where } P \text{ is a pairwise disjoint subset of } E \text{ and has exactly two elements.} \\
\text{b. } & \text{The determiner } \text{neither} \text{ denotes a function which assigns, to each } P, \text{ the family } \{X \subseteq E: P \cap X = \emptyset\}, \text{ where } P \text{ is a pairwise disjoint subset of } E \text{ and has exactly two elements.}
\end{align*}

To illustrate, let us assume a model whose universe is as diagramed in (2), whose set of rings is the set \( \{b, a+c\} \) of Figure 4, and whose set of expensive entities is the set \( \{b, c, b+c\} \) of Figure 7. As the reader will be able to verify, the two sentences in (46) will be false, as \( \llbracket \text{expensive} \rrbracket \) is neither a superset of \( \llbracket \text{rings} \rrbracket \) nor a disjoint of \( \llbracket \text{ring} \rrbracket \).

\[(46)\]

\begin{align*}
\text{a. } & \text{Both rings are expensive.} \\
\text{b. } & \text{Neither ring is expensive.}
\end{align*}

Finally, notice that both identifies the supersets of particular sets while neither identifies the disjoints of particular sets. This means that the

\textsuperscript{14} As in the preceding chapter, we regard this as a semantically well formed expression which is pragmatically ill formed.
dual determiners will still coincide with the appropriate universal and nonexistent quantifiers under the appropriate conditions.

(47) a. $\llbracket \textit{both} \rrbracket (P^*) = \llbracket \textit{every} \rrbracket (P)$ for any pairwise disjoint $P \subseteq E$ such that $|P| = 2$
    b. $\llbracket \textit{neither} \rrbracket (P) = \llbracket \textit{no} \rrbracket (P)$ for any pairwise disjoint $P \subseteq E$
    such that $|P| = 2$

It follows that we can still give a unified account of the illformedness in (29a) and (48a), on the one hand, and in (42a) and (48b), on the other.

(48) a. Both rings look alike.
    b. Neither ring looks alike.

16 The Definite Article: Generic Uses

The proposals presented in this chapter interact in rather interesting ways with the theory of the definite article presented in Chapter 3. Indeed, if the proposals of this and the preceding chapter are taken jointly, they will straightforwardly account for the semantics of definite generics, as we will now detail.

Let us begin by noting that the proposals presented in this chapter do not affect our ability to predict the fact that the definite article carries with it a presupposition of existence. These proposals will provide us, however, with semantically vacuous nouns, and hence with grounds for testing this prediction. As has been pointed out, vacuous nouns will fail to denote sets which have greatest elements. Their combination with the definite article will therefore be predictably undefined. Thus if a noun like unicorn denotes the empty set, then the denotation of a phrase like the unicorn is undefined. This, it seems to us, should be as desired.

Along similar lines, the proposals presented in this chapter do not affect our ability to predict the fact that the singular use of the definite article carries with it a presupposition of uniqueness. In addition, these proposals will provide us with new grounds for testing this prediction. Let us suppose for the sake of illustration that we have a stem ring which denotes the entire universe diagramed in (2). If the interpretation of this stem proceeded relative to a model which abided by the general condition on stems given in (7), then this stem will have the five readings indicated in (8) above. Now, each of these stems will give rise to a different singular noun. These nouns will denote as follows:
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(49) a. \([\text{ring}_1 + \text{SINGULAR}] = \{a, b, c\}\)
b. \([\text{ring}_2 + \text{SINGULAR}] = \{a, b+c\}\)
c. \([\text{ring}_3 + \text{SINGULAR}] = \{b, a+c\}\)
d. \([\text{ring}_4 + \text{SINGULAR}] = \{c, a+b\}\)
e. \([\text{ring}_5 + \text{SINGULAR}] = \{a+b+c\}\)

As can be readily seen, the proposals made in this chapter have provided the definite article with four new potential singular uses. While *the* could only apply before to the set in (49a), it can now apply to the sets in (49b-e) as well.

But the definite article denotes a function which selects the greatest element from any set which has one. Of the five sets in (49), there is only one which has a greatest element: it is (49e). As a consequence of this, the application of *the* to *ring* is defined (in this model) just in case the noun *ring* is taken in the sense defined in (49e). This interpretation runs as follows:

(50) \(\llbracket \text{the ring} \rrbracket = \llbracket \text{the} \rrbracket(\llbracket \text{ring} \rrbracket) = \Gamma(\{a+b+c\}) = a+b+c\)

It should be apparent that the reading in (49e) arose only as a consequence of the proposals advanced in this chapter. If it were not for (7), the noun *ring* would only have the reading defined in (49a), and the denotation of *the ring* would be undefined.

These predictions are strikingly confirmed by the facts. Consider a model which contains a plurality of atomic rings. Suppose we were to interpret a noun phrase like *the ring* relative to such a model. As is generally agreed, our noun phrase will be well formed only as a definite generic. But we have proposed that a kind, the standardly accepted denotation of a definite generic, is the mereological sum of its instances. Such is the only possible denotation *the ring* will receive under the present proposals.

Let us state the foregoing in more general terms. We have proposed that a singular noun denotes a pairwise disjoint set. The definite article may combine meaningfully with a singular noun only if the noun denotes a set with a greatest element. A pairwise disjoint set can have a greatest element only if it is a singleton. The definite article may therefore combine meaningfully with a singular noun just in case the noun denotes a singleton (hence the presupposition of uniqueness). In such a case, the combination of the article and the noun will denote the sole element of the said singleton.

Naturally, this element will be a kind—every element of the universe is one. Yet, kinds are either proper or improper. If the kind denoted by the combination of the definite article and the singular noun
is proper we may say that this combination is a *proper definite generic*; if it is not, we may speak of an *improper definite generic* instead. Alternatively, improper definite generics may be referred to as *proper definite descriptions*, as these are the only ones that Russell (1905) had in mind. As to proper generic descriptions, they can be alternatively referred to as *improper definite descriptions*.

In discussing Russell's theory of descriptions, Moore (1944, 214f) considered sentences like *the heart pumps blood into the arteries, the right hand is apt to be better developed than the left, the triangle is a figure to which Euclid devoted a great deal of attention, the lion is the king of the beasts, and the whale is a mammal*. He then pointed out that 'it is obvious that no part of the meaning of any one of these sentences is (respectively) "at most one object is a heart," "at most one object is a right hand," "at most one object is a triangle," "at most one object is a lion," "at most one object is a whale"'. From this Moore concluded that Russell had given 'a true, and most important, account of at least one use of *the* in the singular', but that even though this use may be by far the commonest, there were other quite common uses to which Russell's account could not apply. To this, Russell (1944, 690) replied

> Mr. Moore points out, quite correctly, that the theory of descriptions does not apply to such sentences as "the whale is a mammal." For this the blame lies in the English language, in which the word "the" is capable of various different meanings.

It should be clear that the definite article is not regarded here as ambiguous between an article of description and an article of genericity. What is ambiguous is not the article, but rather the noun—which seems to be an altogether preferable situation. What is more, this ambiguity is a consequence of an independently motivated consideration, namely the general condition on stems proposed in (7) above.

Things will of course be different with the plural uses of the definite article. A plural noun denotes a mereology, and mereologies are complete. The denotation of a plural noun will therefore contain its sum (if it exists). As a consequence of this, nonvacuous plurals will always have a greatest element, and their combination with the definite article will always be meaningful. As desired, the presupposition of uniqueness will not extend to the plural case.

Evidence for these predictions is not hard to find. Let us suppose a model whose universe contains two or more rings. Consider now the contrasts in (51) and (52).

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(51)  a. The rings are one (in number).
     b. The rings are two (in number).

(52)  a. The ring is one (in number).
     b. ?The ring is/are two (in number).

(51a) is well formed because there is a plural noun rings which denotes a subdomain with one atom, namely \{r\}, where r is the sum of all the rings in the universe. (51b) is well formed because there is at least one plural noun rings which denotes a subdomain with two atoms (the universe was chosen so as to have at least two rings). (52a) is well formed because there is a singular noun ring which again denotes \{r\}. Crucially, (52b) is ill formed because there can be no singleton subdomain with two atoms—and such is what the singular noun ring in (52b) is expected to denote.

Consider next the contrast between (53a) and (53b). Notice that (53b) can make an 'additive' assertion concerning the collective weight of lions. No such assertion can be made by (53a); it simply reports the weight of a single (albeit gargantuan) lion—possibly the generic lion.

(53)  a. The lion weighs 1,000,000 pounds.
     b. The lions weigh 1,000,000 pounds.

This contrast can also be accounted for by our proposals. For, notice that there cannot be more than one lion referred to in (53a). Hence, there cannot be any plurality of lion weights for us to truly add. (53b), on the other hand, may well refer to more than one lion. Hence, only here can a properly additive reading arise.

Or consider the contrasts exhibited by the following sentences relative to a universe which contains a plurality of rings.

(54)  a. The rings look alike.
     b. ?The ring looks alike.

As has been pointed out repeatedly, only nonatomic elements of a subdomain can look alike. But only the subdomain denoted by the stem ring in (54a), not the subdomain denoted by the corresponding stem in (54b), may have nonatoms. Hence, given the semantics of look alike proposed above, only the sentence in (54a), not the sentence in (54b), may be well formed.

Naturally, to account for the contrasts in (51)–(54) we must extend our assumptions about look alike to cardinal adjectives and measure phrases like weigh(s) 1,000,000 pounds. We must thus assume that they all denote functions which assign, to each contextually provided subdomain in the universe, a set of nonatomics thereof. To be more
specific, let us recall that as interpreted above, a cardinal adjective denotes a function over (atomistic) mereologies. When the cardinal adjective is in attributive position, the mereology comes from the adjacent noun. But when the cardinal adjective is in predicative position, as it is in (51) and (52), where does the mereology come from? We assume that it will come from context, which will provide the mereology denoted by the stem of the subject noun.

Consider now (51b). Here two will denote a function which will apply to the (atomistic) mereology denoted by its stem ring. This mereology may have two atoms (we assumed a universe with two or more rings). If it does, the sentence will be well formed (and moreover true). Consider next (52b). Here two will once again denote a function which will apply to the (atomistic) mereology denoted by its stem ring. The result of this application will be the empty set, since the said mereology will not have nonatomic elements (if the subject of (52b) is meaningful, ring must denote a singleton, and hence a mereology with just one atom). As a consequence of this, (52b) will be ill formed, as it asserts that the empty set has a ring as member.

An entirely parallel solution is available for the contrast in (53). For indeed, the 'additive sense' of weigh 1,000,000 pounds can be taken to be a function over nonsingleton mereologies (these are the only ones in which there is something to properly add). If such mereologies can be provided by context, then they will be provided only in cases like (53b), not in cases like (53a), since the singular noun lion of the latter (and hence its stem) must denote a singleton, and hence a mereology with just one atom. As a consequence of this, (52b) will be ill formed, as it asserts that the empty set has a ring as member.

17 More on Generic Uses

Let us consider now a generic statement like (55a) and a universal statement like (55b) relative to a universe which contains a plurality of ballads.

(55) a. I like the ballad.
    b. I like every ballad.

Suppose that one of the ballads in the universe of discourse is the ballad of Gregorio Cortez. It is generally acknowledged that (55a) would not imply (56), as I may like a literary genre without liking all its instances. (55a) thereby contrasts with (55b), which may in fact imply (56).

(56) I like the ballad of Gregorio Cortez.
Facts of this sort are usually adduced in support of the view that generic statements are defeasible: they allow for exceptions. To see that the defeasibility of generics follows from our proposals we begin by observing that the noun ballad in (55a) must denote \{b\}, where \(b\) is the sum of all the atomic ballads in the universe. Now, no such reading is forced on the noun ballad in (55b). Let us suppose then that this noun has its 'primary meaning', namely the set of atomic ballads in the universe. Notice that \(b\) will not be an atomic ballad (we have assumed a universe with a plurality of ballads). Hence something may be true of \(b\) without being true of every atomic ballad of the universe. In particular, something may be true of \(b\) (say, that I like it) without being true of the ballad of Gregorio Cortez.

Similar points can be made with plural generics and plural universals. Consider for instance the statements in (57).

(57) a. The eyes work well together.

b. All eyes work well together.

When taken in its more normal reading, (57a) is not a statement about all eyes, but only about the eyes of each organism. Such a statement is true in the real world. (57b), on the other hand, is primarily a statement about all eyes regardless of their bearers, and is therefore prima facie false in the real world (cf. Dahl 1988, 90).

Our account of the contrast in (57) stems from the assumption that the two instances of the noun eyes in (57) are not synonymous. As we see it, the noun eyes in (57b) denotes the entire domain of \(e\), where \(e\) is the kind constituted by all the eyes in the universe. The noun eyes in (57a), on the other hand, denotes only a proper subdomain of \(e\). It is the subdomain whose atoms are the eyes of each eyed organism (this subdomain will thus have half as many atoms as the first if all organisms have two eyes). Now the contrast in (57) can be seen to follow directly from our proposals. (57a) is indeed a statement about the eyes of each organism; (57b) is the desired statement about all eyes regardless of their bearers. The former may easily be true when the latter is false.

In fact, it follows from our proposals that all plural definites, be they generic or otherwise, may be defeasible, as something may be true of a kind without being true of all its instances. The defeasibility of definite plurals can be further illustrated by the contrast in (58).

(58) a. The Marines invaded Grenada.

b. All Marines invaded Grenada.

For, as Roberts (1986, 147) has pointed out, (58a) may be true although (58b) may be false; not all members of the U.S. Marine Corps
need have gone to Grenada. Contrary to Neale (1990, 45), 'the Fs are Gs' is not equivalent to 'all Fs are Gs and there is [one or] more than one F'.

Now, to say that (SSa) does not entail (56) is to say that there are interpretations in which (55a) does not entail (56). There may well be some interpretations in which (55a) does imply (56). It might be expected that the semantics of generics should be able to say what these interpretations are. It might be thought, for instance, that the semantics of generics should refer to a 'default logic' relative to which one could infer whether or not (55a) entailed (56) in any given model. It seems to us that such expectations demand too much of semantics and too little of pragmatics. General reasons for this view have been provided in Dowty, et al. (1981, 50ff), where it is argued that it is not necessary and perhaps not even possible to provide "a logic which is capable of accounting for all correct inferences made in natural language and which rules out incorrect ones" (cf. Lakoff 1972, 589). Reasons particular to generics have been provided by Geurts (1988, 171f), who argues that "there is a remarkable contrast between the relative simplicity and stability of generic sentences on the one hand and the complexity and dynamic nature of default rules on the other." What a generic sentence denotes and what it implies are two legitimate questions which should be kept distinct.

Let us turn next to the phenomenon of 'proper kind predication'. As noted by Carlson (1978), only (proper) kinds tolerate predicates like widespread, common, fast disappearing, and often intelligent. Such predicates are therefore said to be 'proper kind predicates' and to effect 'proper kind predication'. Now, as we would expect, definite generics tolerate these predicates:16

(59) The owl is widespread/common / fast disappearing / often intelligent / that kind of bird.

Furthermore, the theory of genericity we have advanced suggests a natural constraint on the class of proper kind predicates: a proper kind predicate is an expression which corresponds, whenever used, to a set of proper kinds (or nonatoms) of the universe.

In addition, the present theory of genericity allows for a resolution of the paradox of omnipresence (or at least multipresence) posed by sentences like (60).

(60) The human immuno-deficiency virus is now everywhere.

16 The following examples were provided in Carlson (1978, 276).
For, notice that it seems natural to say that a proper kind will be everywhere (relative to a given space) if every (relevant) location is occupied by an instance of that kind. It follows that (60) does not require any individual to actually occupy every (relevant) location. Indeed, it does not require any individual to actually occupy more than one location.

Before closing this section we should mention some puzzles associated with definite generics. It is hoped that the proposals presented in this work will contribute to their eventual solution. The puzzles have to do with productivity—specifically with the claim that the productivity of definite generics is limited in ways which remain, as of yet, undetermined. Consider for instance the awkwardness of the generic sentences in (61)-(63), which are due to Vendler (1971), Heny (1972), and Carlson (1978), respectively.

(61) ?Euclid described the curve.

(62) a. ?The ruminant has cloven hoofs.
    b. ?The road was first built in Finland.
    c. ?The house has several rooms.
    d. ?The airport is a busy place.

(63) ?The angel plays the harp.

Although we agree with these judgements, we would like to point out that their awkwardness cannot reflect a constraint against the italicized combinations. For consider sentences like the curve is not the shortest distance between two points, the compound four-chambered stomach of the ruminant and the process of rumination have excited the interest of agriculturalists and biologists generally from time immemorial, cars and bicycles should learn to share the road, the angel as a winged creature is not original with Christianity. Although these examples cry out for an explanation of their difference with (61)-(63), they make the case that the awkwardness of (61)-(63) should not be accounted for simply by preventing some nouns from appearing in generic constructions.

The productivity of definite generics is also known to be curtailed when they occur in construction with stage predicates, as parts of assertions which are less than momentous or significant. "On the whole," writes Carlson (1978, 279), "the contexts in which a definite generic may be used where stages are referred to are those contexts which say

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17 See Benzie and Philipson (1957, 7) as well as the very title of Benzie and Philipson (1957).
something momentous or significant about the species as a whole." Thus, as this semanticist observes, "[(64)] lends to the search an epic flavor, as if succeeding in finding a blue-nosed ground squirrel is an event of great significance (it is rarely observed, or thought to be extinct)."

(64) The zoologists are trying to find the blue-nosed ground squirrel.

Similarly, let us suppose that predicates like be available, be walking, and be drunk are true of every policeman in the universe of discourse. If these predicates are closed under sums, then the model would accept ?The policeman is drunk as a true generic sentence. But suppose that our sentence is provided with context which expresses the significance of the original statement. Consider, that is, a sentence like The Freedonian policeman is always available, constantly walking the beat, and drunk when he is off duty. I find no problem in getting a generic reading for this sentence—one under which it may even be true. So definite generics indeed seem to be subject to a 'significance constraint'. Nothing in our account predicts this, however.

To conclude, we will point out that in recent work on generics, Link (1988, 327) proposed "an 'up-arrow' operation ↑ on nouns producing kind-denoting terms; for instance, if TIGER is a one-place predicate denoting the set of tigers, ↑TIGER is a singular term that denotes the kind TIGER". The main purpose of this and the preceding section has been to show that such an up-arrow operation need not be postulated ad hoc, but that it can be instead identified, at least for definite generics, with the greatest function, an operation which exists by virtue of the independently motivated claim that the set of individuals provided by a model must constitute a mereology with the relation of instantiation.

18 Syntactic Denotations

Nothing we have said so far in this chapter forces us to alter the need to provide a syntactic denotation for the definite article. In fact, nothing requires us to alter the actual syntactic denotation assigned to the definite article in Chapter 3. As indicated in the preceding chapter, the said denotation will allow us to attain a unified sentential semantics, to predict the relations between the and every, to account for the 'self description' of definite descriptions (cf. the validity of The ring is a ring), and to distinguish between the two Ns and both Ns.
We should only add, perhaps, that care should be taken in interpreting (65), the prediction made by our proposals concerning the relations between the and every.

(65) If $N$ is a singular noun then $[\text{the } N] = [\text{every } N]$ whenever both terms of the equation are defined.

The prediction holds only of a particular $N$. As a glance at the preceding account of the defeasibility of generics would reveal, (65) fails to hold if different $N$s are involved in each term of the equation.

19 Genericity and Intensionality

We have assumed that semantics operates relative to a model, and that this model provides a set of states of affairs.\(^\text{18}\) To interpret an expression is to assign it a referent in every state of affairs in which the expression is meaningful. Although the semantics of definite generics we have proposed makes no reference to states of affairs, this semantics can be extended in perfectly standard ways so as to assign, to each definite generic, a referent in each state of affairs. It is to this extension that we will now turn.

Let us suppose that nouns denote functions which assign, to each state of affairs, a subset of the universe. Let us furthermore assume that the definite article denotes a function which assigns, to each state of affairs, the greatest function of the universe (the definite article thus denotes a constant function of the set of states of affairs). Let us furthermore assume

(66) The Principle of Intensional Composition

Let $a$ be an expression which denotes a function from states of affairs to functions from a set $X$ to a set $Y$. Let $b$ be an expression which denotes a function from states of affairs to elements of $X$. The denotation of the combination of $a$ and $b$ is a function from states of affairs to elements of $Y$. This function, call it $[[a] /[[b]]$, assigns $[a](w)([[b]](w))$ to each state of affairs $w$.

Equipped with these three assumptions we may now interpret the whale as indicated in (67), where $W_i$ stands for the set of whales in a state of affairs $w_i$ and $i$ is a positive integer (for the sake of illustration we have proceeded relative to a model which provides only denumerably many states of affairs).

\(^{18}\) See Chapter 1.
So if the set of whales coincides with the set of sperm whales in some state of affairs, the *kind* of whales will coincide with the *kind* of sperm whales in that state of affairs. But this does not mean that the *concept* associated to the kind of whales will coincide with the *concept* associated to the kind of sperm whales. For, as indicated in Chapter 2, the concept of a kind is a function from states of affairs to kinds. But two functions may easily agree for some arguments and yet be distinct. Two concepts of kinds may thus easily agree for some states of affairs and yet be distinct.

This means that we can distinguish between different kind concepts which happen to have the same extension in a particular state of affairs. And conversely, we are able to unify different kinds under a single concept, as a single concept may have different instances for different states of affairs (a function may assign different values to different arguments). So the fact that the population of whales has not remained constant over time does not argue against the existence of a single concept corresponding to the kind 'whale'; it only argues against the existence of a single kind 'whale'.

Now, it has been argued by Kripke (1972) and Putnam (1975) that nouns like *dog*, *tiger*, and *whale* are rigid designators: they have the same referent in every state of affairs. The most influential argument to this effect is due to Putnam, and runs as follows. Suppose that a new planet is discovered. This planet is called Twin Earth because it is entirely like Earth except for one thing: the creatures of Twin Earth that look like Earth whales are in fact fish, not mammals. To describe this situation we would say that Twin Earth contains fish that look like whales, not whales that are fish. In the same way, if one describes not another planet in the actual universe but rather another universe with creatures that look like whales but are fish, then one has described a state of affairs with whalelike fish. One has not described a state of affairs in which whales fail to suckle their young. There are no such states of affairs; whales must suckle their young in every state of affairs.

But does this argument establish the claim that a noun like *whale* denotes rigidly? Not really. It only establishes that *whale* is not a completely flexible designator—or that there are limits to what the
reference of this noun can be in each state of affairs (creatures that suckle their young). Notice that this refinement of the position of Kripke and Putnam solves the problem attributed to Donnellan by Schwartz (1977, 37f). If nouns like *whale* denote rigidly, how is it possible that states of affairs may differ as to the set of whales they contain? Indeed, how can there be two states of affairs whose sets of whales are entirely disjoint? Our solution is that the denotation of *whale* may vary, within certain limits, from one state of affairs to another. As argued by Kripke and Putnam, the limits of variation would be set by the nature of whales—say their DNA.¹⁹

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¹⁹ It has been claimed by Dahl (1975, 108) that indefinite generic noun phrases "always involve quantification over possible objects rather than over actual ones". It should be noticed that if this view carries over to definite generics, then excessively strong truth conditions are expected of generic sentences. For, consider once again the sentence *Turing invented the computer*. What this sentence would be asserting is that Turing invented the computer in every state of affairs containing the computer. But then the sentence would have to be false, as it is logically possible that someone other than Turing could have invented the computer. Or consider *The dodo is extinct*. For this sentence to be true, every possible dodo (species) must be extinct. But the sentence is intuitively true even though it is not valid.
1 Introduction

We began our discussion of countability by acknowledging, with Jespersen (1924, 198), the existence of expressions which 'call up the idea of some definite thing with a certain shape or precise limits.' To account for this fact, Jespersen assumed a 'world of countables' which was inhabited by entities like houses, horses, days, miles, sounds, words, crimes, plans, and mistakes. It will be recalled that we construed this world as a subset of the universe. It was the set of kinds which were called 'atomistic' since they were entirely constituted by atoms of the universe.

But as Jespersen would have it, languages contain also expressions which fail to call up the idea of 'some definite thing with a certain shape or precise limits'. To account for this fact, the Danish grammarian assumed a 'world of uncountables' which was inhabited by entities like silver, quicksilver, water, butter, gas, air, leisure, music, traffic, success, tact, common sense, satisfaction, admiration, refinement, restlessness, justice, safety, and constancy. In the view of this chapter, the world of uncountables is again a subset of the universe. It is the set of kinds which could be called 'atomless' insofar as they are instantiated by no atom of the universe.

To gauge the descriptive power of this view, let us recall that an atom of a mereology is, by definition, an element which has no proper subelements. Let us moreover say that a mereology is atomless just in case none of its elements has an atomic subelement. More formally, let $E$ be a set and $\leq$ be a binary relation which jointly constitute a mereology. This mereology is atomless if and only if no $x \in E$ is such that there is an atom $a$ of the mereology such that $a \leq x$. A universe is therefore atomless if and only if no kind of the universe has an atom as instance.

Consider for example the mereology of solids mentioned in Chapter 2. This was the mereology whose domain was the set of solids in space and whose relation was the 'part of' relation appropriate thereto. The
interest of this mereology in the present context stems from the fact that every solid will contain another solid as proper part. This means that there will be no minimally inclusive solids, and that the mereology of solids will therefore be atomless. It should be clear that the empty mereology is atomless (the empty mereology is therefore both atomistic and atomless!). But if an atomless mereology has any elements whatsoever, then it will have infinitely many. Naturally, not every mereology which is infinite will be atomless.

The distinction between countable and uncountable expressions drawn by Jespersen is amply justified by the large number of linguistic phenomena it governs. Thus, as is well known, only countables may be properly pluralized, numbered, ordered, described in terms of size, described in terms of shape, and quantified in individual terms. In fact, the distinction between countable and uncountable expressions can even govern inflectional morphology. Thus, according to Jespersen (1924, 240), some dialects of Southwestern England refer to 'full shapen things' as he (acc. en) and are specified by theāse (a proximal determiner) or by thik (a nonproximal determiner); 'unshapen quantities of stuff', however, are referred to as it, and are specified either by this (a proximal determiner) or by that (a nonproximal determiner):

We say of a tree 'he's a-cut down', 'John vell'd en', but of water we should say 'It's a-dried up' [...] If a woman had a piece of cloth she might say "This cloth is wide enough vor theāse teāble" since, as long as it is unshapen into a tablecloth, it is impersonal [= uncountable]; but as soon as she may have made it up into a tablecloth, it belongs to the personal [= countable] class, and then we should say of it: 'Theāse or thik cloth do belong to theēse or thik teāble.' [...] If a right-speaking Dorset man were to say 'theāse stwone' I should understand he meant a whole shapen stone, whereas 'this stwone' would mean a lot of broken stone. Of a brick bat he would say 'Teāke en up.' Of a lot of brick-rubbish, 'Teāke it up.' 'Thik ground' would mean a field, but 'That ground' a piece of ground (Barnes 1886, 17f).

Within Germanic, similar facts have been documented in the West Jutland dialects of Denmark (cf. Haugen 1976, 288, 371). And in the Romance family of languages, entirely parallel phenomena can be found in the Northcentral dialects of Spain (cf. Penny 1970), and in the Centromeridional dialects of Italy (cf. Rohlfs 1968, §§419, 456, 494).

**2 Uncountable Stems**

To account for the interpretation of uncountable stems we will once again invoke the domains (or principal ideals) in a universe. It will be

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1 See for instance the works in Pelletier (1979).
recalled that the domain of a kind in a universe is the set of instances of that kind in that universe. More formally, let $E$ be a universe and let $k \in E$. The domain of $k$ in $E$ is the set $E \setminus k$, where

$$E \setminus k = \{x \in E : x \leq k\}$$

To illustrate this notion in an atomless universe, let us begin by letting $E$ be the set of portions of matter in the world and letting $\leq$ be the binary relation which every portion of matter bears to any portion of matter which is at least as inclusive as the first. It should be clear that $E$ and $\leq$ jointly constitute a mereology (we assume that portions of matter can have spatially disconnected parts). But let us suppose now, contrary to modern science, that matter is infinitely divisible. It follows that $E$ and $\leq$ jointly constitute an atomless mereology. Suppose furthermore that $\leq$ is the relation of instantiation. To be a portion is now to be an instance, and $E$ can then be an atomless universe provided by a model. A kind in this universe would therefore be both the sum of its instances and the sum of its portions (see Chapter 2).

Let us say now that our universe contains 'winekind', by which we mean the kind constituted by all the portions of wine in the universe. If we call winekind $w$, the domain of winekind in $E$ will be the set $E \setminus w = \{x \in E : x \leq w\}$, which is the set of portions of wine (or, equivalently, the set of instances of winekind). Naturally, each element of our universe will have a domain. In fact, each element of the universe will have a different domain.

Incidentally, if the mereology of solids could be the (structured) set $E$ of individuals provided by a model, then the domain of a solid $s$ would be $\{x \in E : x \leq s\}$, which is the set of solids which are contained in $s$. Can the mereology of solids be the structured set of individuals provided by a model? Only if to be part of a solid is to be an instance of a solid. Although this assumption seemed reasonable for the mereology of matter, this is not so for the mereology of solids. Interestingly, however, it seems rather useful to think of the mereology of matter, which will play a fundamental role in the sequel, in terms of the mereology of solids.

Be that as it may, it will be recalled that every domain in a universe is a mereology in its own right. As such it may be atomless, as $E \setminus w$ happens to be. Moreover, every domain in a universe will have submereologies.$^2$ In fact every domain will have three possibly distinct

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$^2$ It will be recalled that a subset (of a mereology) which is closed under both sum and complementation was said to be a submereology (of the mereology). See Chapter 4, Section 2.
submereologies: the empty submereology, the full submereology, and the universal submereology. In the case of $E \models w$, the empty submereology is $\emptyset$, which is the empty subset of the set $E \models w$ of all the portions of wine. The full submereology is $E \models w$, which is the set of all the portions of wine proper. The universal submereology is $\{w\}$, which is the set whose only member is winekind. Now, of these submereologies, the universal submereology is atomistic, the full submereology is atomless, and the empty submereology is the empty mereology, so it is both atomistic and atomless.

But let us recall now that the subdomains of an individual are, precisely, the submereologies of the domain of that individual. Let us also bear in mind that every subdomain in a universe will constitute a mereology in its own right (when taken in conjunction with the relation of instantiation of the universe). Naturally, such a mereology may be atomless. We may therefore speak coherently of atomless subdomains. The interpretation of uncountable stems may now be constrained by proposing the following condition on the admissibility of models.

(2) **Uncountable Stems**

Every uncountable stem denotes an atomless subdomain in the universe.

Thus, if $wine$ is an uncountable stem, then it may denote as indicated in (3) relative to a model whose universe $E$ is the set of portions of matter discussed above.³

(3) $\langle wine \rangle = E \models w = \{x \in E: x$ is an instance of $w\} = \{x \in E: x$ is a portion of wine$\}$

But $wine$ will indeed have further denotations in our model. Let us suppose that all the wine in our universe $E$ is collected in one vast barrel. The wine in this barrel will obviously have a northern half and a southern half. Just as obviously, each of these halves will have an eastern half and a western half. But each of these quadrants will have its own halves—and so on. Since we are operating relative to a universe wherein matter is infinitely divisible, every wedge shaped portion of wine will properly contain further wedge shaped portions of wine.

Let us construct now the set formed by all of the wedge shaped portions of wine *and their sums*. This set will be contained, as a subset,
in the set of all portions of wine in the model. But the set we have constructed is closed under sum and under complementation as taken in \( E \setminus w \). We have therefore constructed a submereology of \( E \setminus w \) or, what amounts to the same thing, a subdomain of \( w \).

Naturally, the subdomain of \( w \) we have just constructed is atomless (as we have seen, the infinite divisibility of matter in our model guarantees that wedge shaped portions of wine would always contain further wedge shaped portions of wine). Yet the subdomain we have constructed is not the full subdomain of \( w \), as there are portions of wine which will be absent from this subdomain (consider for instance the top and the bottom halves of the wine in our barrel!).

Let us now recall the general condition on nominal stems repeated here in (4). As pointed out in the preceding chapter, the effect of this condition is to assign a set of 'secondary meanings' to nominal stems which have an appropriate 'primary meaning'. To be more specific, the general condition on nominal stems will take any nominal stem which may denote the domain of a kind and make sure that this stem will also denote any (nonempty) subdomain of that kind.\(^4\)

\[(4)\quad \textbf{Nominal Stems}\]

Let \( E \setminus k \) be the domain of some individual \( k \) in some universe \( E \).
Let \( P \) be a nonempty subdomain of \( k \) in \( E \). If there is a nominal stem which denotes \( E \setminus k \), then there is another nominal stem which is homophonous to the first but denotes \( P \).

Now, it will be noticed that the general condition on nominal stems applies to nominal stems regardless of countability. So let us suppose that the uncountable stem \( \text{wine} \) denotes as indicated in (3). By (4), a stem \( \text{wine} \) will also denote any nonempty subdomain of \( w \). It will therefore denote the set generated by the wedge shaped portions of wine which we have constructed above. As will be seen below, this denotational flexibility of uncountable stems should be as desired.

Finally, it should be recalled that only infinite mereologies can be atomless. But since every subdomain constitutes a mereology, (2) entails that every nonvacuous, uncountable, stem denotes an infinite set. In fact, it can be shown that every such stem denotes a nondenumerable

\(^4\) The general condition on nominal stems is a condition on the admissibility of models. To make sure that a model has the required senses for its stems, the condition may proceed either \emph{synthetically} (by providing the models with the required secondary meanings) or \emph{analytically} (by filtering out the models which fail to provide the required secondary meanings). To simplify the presentation of models the semanticist may want to interpret the said condition as a synthetic rather than as an analytic device.
set. This means that the question raised in van Benthem (1986, 7) as to whether natural languages may be interpreted with finite models must be answered in the negative in the present setting. If the proposals of this chapter are adequate, infinite models will be called for by the semantics of uncountability. As argued in van Deemter (1984) on entirely independent grounds, the principled restriction to finite universes must be abandoned.

3 Secondarily Countable Stems

It follows from (4) that wine is at least two ways ambiguous. It may denote either the set of all portions of wine in the universe, or else the set generated by the wedge shaped portions of wine we have constructed. But this stem will still have further senses relative to the models we are considering. Consider first \{w\}. This is the universal subdomain of w. Clearly, this subdomain is not empty. The general condition on nominal stems will thus make sure that this set is yet another denotation of our stem. Now, it should not escape the reader that \{w\} is an atomistic subdomain of w. As a consequence of this, \{w\} cannot be the denotation of an uncountable stem; \{w\} will rather be the denotation of a countable homophone of the stems wine we have been considering so far.

But wine may have far more interesting countable senses in our models. Let us assume that there are three kinds of wine in our universe: the red, the white, and the rosé. Naturally, these kinds will constitute a subset of the domain of wine. In fact, they will constitute a pairwise disjoint subset of the domain of wine. Moreover, these three kinds, call them x, y, z, will constitute the entire kind 'wine'. They will therefore serve as the atoms of the atomistic subdomain of w which we may diagram as in (5).

By the general condition on nominal stems formulated in (4), this subdomain is yet another countable sense of wine. Needless to say, the mereological structure of this subdomain is indistinguishable from the mereological structure of our paradigmatic atomistic universes.

\[\text{For, consider the set of wedge shaped portions of wine we have constructed above. There exists a set of such wedges which is both pairwise disjoint and countably infinite. Since the set is pairwise disjoint, there will be as many sums as there are nonempty subsets. But since the set is countably infinite, it will have nondenumerably many (nonempty) subsets.}\]

\[\text{Pelletier and Schubert (1989, 379ff) have argued that the set of kinds of any given substance constitutes a join semilattice when taken in conjunction with the relation 'is a kind of'.}\]
Cast in more general terms, let $k$ be an atomless kind of a universe $E$. Consider the domain $E \upharpoonright k$ of this kind. $E \upharpoonright k$ is a possible denotation for an uncountable stem $N$. Let us say now that $P$ is a pairwise disjoint subset of $E \upharpoonright k$. Let us say also that the elements of $P$ constitute $k$. But consider now $P^*$, the set of all sums over $P$. $P^*$ is an atomistic subdomain of $k$ (and that $P$ is the set of atoms thereof). Since $P^*$ will be nonempty, the general condition on nominal stems will ensure that $P^*$ is a possible denotation of a countable stem which is homophonous to $N$. It should be clear that the preceding applies to any uncountable stem $N$ which denotes anything but the empty set. Hence we will be able to deduce below the observation that every uncountable stem may be used as a countable stem. In fact, we will be able to derive the common observation that any countable counterpart of a primarily uncountable stem ‘will refer to kinds’ (more precisely, we will be able to predict that such a stem will denote an atomistic mereology of kinds which constitutes the same kind as the set denoted by its uncountable counterpart).

4 The Materialization Function

Let us say now that there is gold in our universe $E$ of portions of matter. The stem gold will therefore denote as indicated in (6), where $g$ stands for the kind constituted by all the portions of gold in $E$.

\[(6) \quad [\text{gold}] = E \upharpoonright g = \{x \in E : x \text{ is an instance of } g\} = \{x \in E : x \text{ is a portion of gold}\}\]

But let us say now that all the gold in our universe has been cast into three rings $a$, $b$, $c$. Let $m(a)$, $m(b)$, $m(c)$ represent the gold (or the matter) contained in $a$, $b$, $c$ respectively. It follows from our assumptions about the universe that $\{m(a), m(b), m(c)\}$ is a pairwise disjoint subset of $E \upharpoonright g$ which constitutes $g$. As a consequence of this, the closure of
\{m(a), m(b), m(c)\} under sum is an atomistic subdomain of the kind ‘gold’; its structure is as depicted below.

(7)

\[
\begin{array}{c}
m(a)+m(b)+m(c) \\
\quad m(a)+m(b) \quad m(a)+m(c) \quad m(b)+m(c) \\
\quad \quad m(a) \quad m(b) \quad m(c)
\end{array}
\]

It will be clear that the mereological structure of the subdomain in (7) is indistinguishable from the structures of certain sets of rings employed in our discussion of countability. Yet, the two structures will articulate entirely different sets. One structure is defined on rings, whereas the other is defined on their contents. That languages distinguish rings from the gold that makes them up is revealed by sentences like (8), where contradictory properties are ascribed to a ring and to the gold which makes it up.\(^7\)

(8) My new ring is made of old gold.

Since opposites have been traditionally defined as predicates which cannot be true of the same object (in the same sense, at the same time), my ring and the gold contained therein are different objects. It follows that we can represent the fact that languages may distinguish between two distinct individuals which nevertheless occupy the same space (in the same sense, at the same time).

The notation ‘\(m(x)\)’ we have employed in (7) suggests the existence of a ‘materialization function’—one which assigns, to each atomistic kind, the ‘matter’ which makes it up. Let us now be explicit and propose that, in order to be fit for describing the semantics of natural languages, a model must provide a function which assigns, to each atomistic kind of its universe, an atomless kind thereof. In addition, we will propose that this function, called the materialization function \(m\), preserves sum and complementation. This means that \(m\) must satisfy the equations in (9), where \(a\) and \(b\) are atomistic kinds of the universe.

(9) \[m(a+b) = m(a) + m(b)\]
\[m(a') = m(a)\]

\(^7\) See Link (1983, 307).
If we furthermore propose that the materialization function is injective, then the atomistic domain will have a structurally identical counterpart in the atomless domain.\textsuperscript{8} Technically, the atomless domain will contain an \textit{isomorphic image} of the atomistic domain.\textsuperscript{9}

The materialization function is the mathematical expression of the colorful Universal Grinder, a hypothetical contraption envisaged by F. J. Pelletier:

Consider a machine, the "universal grinder". This machine is rather like a meat grinder in that one introduces something into one end, the grinder chops and grinds it up into a homogeneous mass and spews it onto the floor from its other end. The difference between the universal grinder and a meat grinder is that the universal grinder's machinery allows it to chop up any object no matter how large, no matter how small, no matter how soft, no matter how hard (Pelletier 1975, 6f).

Notice that we do not wish to restrict \( m \) to material individuals of the universe. The materialization function is a "universal grinder" of all atomistic kinds. We may therefore speak of the matter (or better the makeup) of personalities or events.

Our interest in the materialization function stems from the need to postulate the following semantic condition. It stipulates that every appropriate countable stem has a homophonous uncountable counterpart which denotes the portions of matter contained in the makeup of the kind whose domain was denoted by the countable stem.

\begin{equation}
\text{(10) Uncountable Doubling}
\end{equation}

If there is a nominal stem \( N \) which denotes the domain of an atomistic kind \( k \), then there is a nominal stem which is homophonous to \( N \) but denotes the domain of \( m(k) \) instead.

The condition in (10) thus expresses the generalization, made for instance in Allan (1980, 547), that countables can double as uncountables. What is more, the condition provides us with a procedure for generating uncountable senses given countable senses.

The worth of (10) can be illustrated by the interpretation of sentences like (11). For, notice that this sentence is normally taken to

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\textsuperscript{8} A function \( f \) from a set \( A \) to a set \( B \) is \textit{injective} if and only if no element of \( B \) is assigned by \( f \) to more than one element of \( A \).

\textsuperscript{9} This image will furthermore be a subdomain of some atomless kind. A generalization of the materialization function was proposed in a somewhat different setting by Link (1983, 313f).
assert (12a) rather than (12b), as the latter would be contradictory if \( r \) is the atom which *this ring* would denote.\(^{10}\)

(11) This ring is gold.

(12) a. \( m(r) \in \{ x \in E : x \text{ is a portion of gold} \} \)
   b. \( r \in \{ x \in E : x \text{ is a portion of gold} \} \)

But the simplest way to allow for the normal interpretation would be to assume a countable stem *ring\(_1\)* and rely on (10) for the creation of an uncountable stem *ring\(_2\)*. The stem thus created will denote the set of portions of rings in the universe. One of these portions should be in the form of a ring. When uttered under normal conditions, (11) is only about one such portion, not about the ring into which it is cast. Normally, then, (11) will contain not *ring\(_1\)*, but rather *ring\(_2\)* (or in any event a stem derived therefrom by (4), the general condition on nominal stems).

But, is the materialization function really injective? Consider the case of a rope and the hammock entirely woven therefrom (cf. Burge 1975, 202). It would seem that both are made up of the same fibrous material. Yet, they should not coincide. The materialization function would then map two different atomistic objects onto the same atomless object. It follows from these considerations that the atomistic domain would not have a structurally identical counterpart in the atomless domain. It would only have a structurally analogous counterpart therein. Technically, the atomistic domain would contain an homomorphic image of the atomless domain.

Alternatively, we may wish to retain the claim that the materialization function is injective. The said rope and the said hammock would then have to map into different makeups which simply happen to coincide in space and time. To argue for this view notice that the hammock and the rope would not have the same economic value; the former would have an added value issuing from the weaving it involved. If they were to be appraised in bulk, a weight of hammock would be worth more than the same weight of the rope used to weave it. The materialization function would then map the rope and the hammock into what is being appraised in bulk in each case.

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\(^{10}\) "For on the one hand," writes Aristotle, "whenever a formation is completed, we do not call it what it is made of: we do not call the statue 'bronze' or the candle 'wax' or the bed 'wood', but we use a derived expression and call them 'of bronze', 'waxen', and 'wooden' respectively ... [But] we speak of the particular fluid or hot substance as being bronze" (*Physics* VII 3 245b 10-17; cp. *Metaphysics* VII 7 103a 5-23, cited by Sharvy 1983, 232).
The issue of whether two distinct atomless entities may occupy the same space at the same time arises also with sentences like *This snow is young, but the water that makes it up is old*, noticed in Bach (1986). This sentence seems to require that the materialization function map one atomless kind (the snow) onto another (the water making it up). The sentence thus shows that the materialization function cannot be restricted to atomistic kinds, but must apply to atomless kinds as well. In addition, it shows that when the function thus extended applies to atomless kinds, it cannot coincide with the trivial identity function, as proposed in Link (1983). The materialization function introduced in this section must therefore be generalized in nontrivial ways if it is to be part of a descriptively adequate semantics of English. It seems clear, though, that two distinct atomless entities (of a universe of discourse) may occupy the same space at the same time.

5 The Reversibility of Countability

When taken together, the conditions in (4) and (10) imply that every (primary) countable stem will have an uncountable counterpart and, conversely, that every uncountable stem will have a (secondary) countable counterpart. Judging from the literature, \(^{11}\) this would seem to be as desired. As Pelletier and Schubert (1989, 345) put it, "every noun—even hole and pore—sometimes occurs in noun phrases which we would intuitively call +mass. And every noun sometimes occurs in noun phrases we would intuitively call +count."

To account for this dual nature of nominal expressions, we will specify nominal stems in the lexicon as countable or uncountable according to the 'countability preference' of the stems (following Allan (1980), the countability preference of a stem will be computed from the ease with which it may appear in unambiguously countable and unambiguously uncountable environments). \(^{12}\) We will then rely on the conditions in (4) and (10) to complete our interpretation of nominal stems.

Moved by "the ability of English words to be used as EITHER count or mass nouns," Weinreich (1966, 435) considered the possibility of creating separate countable and uncountable entries for each noun in the lexicon. In the view of this grammarian, the possibility had to be

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\(^{11}\) See for instance Quine (1960, 91), Gleason (1965, 137), Weinreich (1966, 435), Pelletier (1975, 6), Allan (1980, 547), and Bunt (1985, 11).

\(^{12}\) Before Allan (1980), Ware (1975, 21) observed that "most nouns can have a role as either a mass noun or as a count noun [...] they all have their double lives, but one is more dominant than the other."
rejected on two grounds. First it led to a doubling of lexical stipulations. Second, it failed to show that "some nouns are more countable than others." As the reader will agree, the strategy proposed in this section avoids these two shortcomings because it does not enter each stem twice in the lexicon; the lexicon will only register the countability preference of each noun (stem). We may then rely on (4) and (10) to generate the remaining senses of our stems and to express the systematic relations which hold among these senses.

6 The Individuation of Reference

Countable stems have been claimed to differ from uncountable stems in that only the former provide a criterion for the individuation of their reference. Thus, as he discussed general terms, Quine pointed out that

To learn 'apple' it is not sufficient to learn how much of what goes on [sic] counts as apple; we must learn how much counts as an apple, and how much as another. Such terms possess built-in modes, however arbitrary, of dividing their reference [...] consider 'shoe', 'pair of shoes', and 'footwear': all three range over the same [...] stuff, and differ from one another solely in that two of them divide their reference differently, and the third none at all (cf. Quine 1960, 91).

But it should be clear that the proposals found under (2) above and under (5) in the preceding chapter allow countable stems to provide a straightforward criterion for the individuation of their reference (the individuals of a countable stem denotation are its atoms) but preclude uncountable stems from providing any such criterion (uncountable stem denotations are atomless). As Krifka (1987, 10) would have it, count nouns have a built in reference to their natural units.

Let us grant now that the proposals advanced in this study distinguish countables from uncountables in terms of the individuation of their reference. Does this distinction between countables and uncountables exist? Soon after he distinguished countables from uncountables in terms of the individuation of their reference, Quine introduced a hedge:

In general a mass term in predicative position may be viewed as a general term which is true of each portion of the stuff in question, excluding only the parts too small to count. Thus 'water' and 'sugar' in the role of general terms, are true of each part of the world's water or sugar, down to single molecules but not to atoms; and 'furniture', in the role of a general term, is true of each part of the

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13 The question thus arises as to whether standard usage should be countered so that atoms should be regarded as the only true individuals of a model. To avoid confusion, however, we will refrain from doing so and retain the notion of individuals set forth in Chapter 1 instead.
world's furniture down to single chairs but not to legs and spindles (cf. Quine 1960, 98, emphasis supplied).

But by excluding "the parts too small to count" from the denotation of an uncountable, Quine takes with one hand the distinction he gave us with the other. For how is the learning of apple now different from the learning of water, sugar, or furniture? Learning the meaning of any of these terms would require learning what counts as one and what counts as many.

But aside from blurring the distinction between countables and uncountables, Quine's hedge overestimates the influence of science on language. To claim that water and sugar are true of entities "down to single molecules but not to atoms" is to claim that the meaning of water and sugar changed once the molecular structure of water and sugar was discovered. As has been pointed out, however, "the development of the molecular conception of the structure of matter did not change the way in which the word water was used" (cf. McCawley 1981, 434). In fact, as will be shown below, the grammatical behavior of water, sugar, and furniture is entirely like that of space, courage, and orderliness, uncountables for which one would be hard put to find minimal parts. To distinguish between uncountables in this way is to do violence to language by positing more distinctions than are called for.

As we see it, the development of the molecular theory of matter changed only the scientific use of terms like water and sugar; it did not touch the ordinary use of these terms. In our view, the change was the replacement of terms like water and sugar by terms like molecule of water, and molecule of sugar, the interpretation of which proceeded relative to models which are fit for the scientific description of nature but not for the scientific description of natural languages.

Incidentally, it is interesting to note that the change in the scientific use of uncountables was not the reanalysis of these terms as countables (common chemical practice does not involve the use of a water to refer to a molecule of water). The specification of water as an uncountable in ordinary language seems to be too much for science to contend with. What we find in common chemical parlance instead is the use expressions like a gas or three gases to refer to a kind of gas or to three kinds of gases—exactly what we would expect from our proposals (cf. Section 9 below).

Be that as it may, the proposals made in this study are incompatible with what has been called the minimal parts hypothesis (cf. Bunt 1985, 45). According to this hypothesis, for each uncountable stem \(N\) there is a specific minimal size that parts of its referent may have in order to be
As far as the present study is concerned, quite the opposite is true, as our claims entail that for no uncountable stem \( N \) will there be a specific minimal size that parts of its referent may have in order to be \( N \).

But the rejection of the minimal parts hypothesis does not lead (and has not led) necessarily to the interpretation of an uncountable stem in terms of an atomless structure; uncountable stems might be *indeterminate* with respect to the hypothesis (countables, on the other hand, would be determinate with respect to a minimal parts hypothesis; the hypothesis would be true of them). Thus, according to McCawley (1981, 434), uncountables refer to entities "which need not have any minimal parts in the way a kangaroo (perhaps better, a set consisting of a single kangaroo) is a minimal part of the set of all kangaroos." For Bunt (1985, 46), uncountables refer to entities related as parts are to wholes but "without making any commitments concerning the existence of minimal parts." As we will soon see, similar considerations can be found in Link (1983, 308).

As we see it, to regard uncountables as indeterminate with respect to individuation makes for an unnecessary weakening of the theory. As has been pointed out above (and will be shown below), the linguistic behavior of uncountables with allegedly minimal parts is entirely like that of uncountables without the said parts. Their linguistic behavior is not that of countables.

7 Some Comparisons

It would indeed be impractical for us to attempt now a complete survey of the formal theories of uncountability which have appeared since the seminal discussion of this topic in Quine (1960, §§19-20). Yet, our work cannot proceed without mentioning the best known of these theories—namely the one found in Link (1983)—and referring the reader to Pelletier and Schubert (1989) for an excellent survey of the literature.

As mentioned at the outset of this study, Link proposed that the set of individuals provided by an admissible model must have the structure of a boolean algebra which is both complete and atomic. In addition Link required that some of the atoms of this algebra have a further structure (namely that of a complete join semilattice), and that this structure be a homomorphic image of the positive portion of the boolean algebra which articulates the entire set of individuals. The reader who is interested in gaining a full grasp of these proposals is referred to Appendix A, where the boolean theory of individuals presented in Link (1983) is made accessible to anyone who controls the rudiments of set theory. For our present purposes, however, the crucial aspect of this
The Semantics of Uncountability

proposal lies with the set of atoms which constitutes the said semilattice, as every uncountable noun is taken to denote a subset of this set.

Now, it might seem perverse to have atoms constitute the denotation of uncountables. Yet, it should be borne in mind that the said atoms are required to constitute a complete join semilattice. As pointed out in Appendix A, this semilattice will in fact turn out to be a mereology. But this mereology may or may not be atomistic. So the sets denoted by uncountables are constituted by elements of a mereology which is not necessarily atomistic. As to the sets denoted by countables, they are required to select their denotations from the atoms which do not constitute the semilattice. So for Link (1983), the difference between countables and uncountables turns on this. Countables denote sets of necessarily atomistic elements, whereas uncountables denote sets of possibly atomistic elements. Countables thus denote sets which must have minimal elements; uncountables denote sets which may or may not have minimal elements.

As indicated in the preceding section, allowing uncountables to be indeterminate with respect to individuation represents a weakening of the theory. Now we see that it furthermore represents a complication thereof. For, by claiming that uncountables denote positively atomless sets we may dispense altogether of the secondary structure (the complete join semilattice) imposed on the domain of individuals provided by an admissible model.

8 Number Inflection

It will be recalled that a countable noun is the combination of a countable stem and a number inflection. But what is an uncountable noun? Let us assume by parity of argument that an uncountable noun is the combination of an uncountable stem and a number inflection. Let us moreover suppose that number inflections denote functions over atomistic subdomains in the universe (see the conditions on number inflections advanced in the preceding chapter). But uncountable stems denote atomless subdomains instead. It follows that if the denotation of an uncountable stem inflected for number is to be governed by the Functional Principle, the uncountable stem must denote the empty set (as pointed out above, the empty set is the only subdomain which is both atomistic and atomless). But the functions denoted by the number inflections are such that they map the empty set onto itself. The only possible denotation for an uncountable noun would therefore be the empty set itself:
(13) a. \([\text{wine} + \text{SINGULAR}] = [\text{SINGULAR}](\text{wine}) = \text{SINGULAR}(\emptyset) = \emptyset\)

b. \([\text{wine} + \text{PLURAL}] = [\text{PLURAL}](\text{wine}) = \text{PLURAL}(\emptyset) = \emptyset\)

But this is a patently false prediction, as uncountable nouns do not have to denote the empty set in every admissible model; uncountable nouns are not contradictory!

9 Uncountable Nouns

To avoid this prediction, we will refrain from assuming that uncountable nouns contain number inflections. We will instead assume that an uncountable noun is an uncountable stem which remains simply uninflected for number. Cast in positive terms, uncountable nouns come in what Jespersen (1954, II, §§5.51ff) wants to call the neutral number, by which he means "a form of number which is neither definitely singular nor plural, which therefore leaves the category of number open or undetermined." Incidentally, the pertinence of such a 'form of number' for uncountables did not escape Jespersen:

The want of a common number form (i.e. a form that disregards the distinction between singular and plural) is sometimes felt, but usually the only way to satisfy it is through such clumsy devices as "a star or two," "one or more stars," "some word or words missing here," [...] In an ideal language constructed on purely logical principles a form which implied neither singular nor plural would be even more called for when we left the world of countables [...] and got to the world of uncountables (cf. Jespersen 1924, 198).

As we see it, natural languages are indeed "ideal languages constructed on purely logical principles"—at least in this respect.

One immediate consequence of the necessarily uninflected nature of uncountable nouns is the often noted fact that true plurality is incompatible with true uncountability (cf. for instance Quirk et al. 1985, §5.75). Notice that this incompatibility is not countered by nouns like oats, clothes, news, molasses. As it is readily accepted, these nouns are pluralia tanta, their forced plurality thus pertaining only to form, not to content. As a consequence of this, the nouns in question do not exhibit true (or semantic) plurality.

Furthermore, the incompatibility of true plurality and true uncountability is not countered by nouns like wines, waters, teas, and golds. As is well known, such nouns do not involve true uncountability. Rather, they contain strictly countable stems. Each of these stems denotes the atomistic subdomain whose atoms constitute a pairwise disjoint set of kinds of a particular stuff—be it wine, water, tea, or
But the existence of such countable senses of a primarily uncountable stem is actually predicted by the general condition on nominal stems given in (4) above. Since this prediction is borne by the facts, the generalized setting developed in Chapter 4 receives further support from the semantics of uncountability.

To illustrate, let us assume with (3) that an uncountable stem *wine* denotes the atomless domain constituted by all the instances of the kind 'wine'. Let us moreover assume that there are three disjoint kinds of wine in our universe which are constitutive of the entire kind 'wine': the red, the white, and the rosé. It was noted that these three kinds will be the atoms of an atomistic subdomain of the kind 'wine'. This subdomain was the one diagramed in (5) above. But by the general condition of stems given in (4), this subdomain will be a possible reading of a countable stem *wine*. This stem will combine with the singular inflection to denote as indicated in (14a). The stem will combine with the plural inflection to denote as indicated in (14b).

(14) a. \[ ([\text{wine} + \text{SINGULAR}]) = ([\text{SINGULAR}])([\text{wine}]) = \{x, y, z\} \]
   b. \[ ([\text{wine} + \text{PLURAL}]) = ([\text{PLURAL}])([\text{wine}]) = \{x, y, z, x+y, x+z, y+z, x+y+z\} \]

Incidentally, the tolerance of number inflection displayed by the countable noun *wine* should be compared with the intolerance displayed in this respect by the uncountable stem *furniture*. In spite of the impression that furniture might have the individual pieces of furniture as minimal parts, the uncountable plural *furnitures* is ill formed. The countable plural *furnitures* is, of course, well formed, but does not denote the atomistic domain having the said pieces as atoms. At best, it will denote the atomistic subdomain having the material of the said pieces as atoms. More commonly, however, it will denote an atomistic subdomain having kinds of furniture (e.g. modular furniture, period furniture, Scandinavian furniture, etc.) as atoms.

10 Cumulative Reference

We should now observe that (2) stipulates that uncountable stems denote subdomains in a universe. But subdomains are complete sets.15

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14 We ignore in this study those uses of *wines* and *waters* found for instance in bars or restaurants, where the beverages thus named are served in standardized quantities. In such contexts, the stems *wine* and *water* have simply developed countable senses corresponding to 'standard unit of wine' or 'standard unit of water'—in strict accordance with the proposals advocated here.

15 See the previous sections on cumulative reference.
Uncountable stems thus have cumulative reference. Since uncountable nouns are just uncountable stems, they too will have cumulative reference. And, as is well known, this prediction is borne by the facts. Sentences like (15a) and (15b) imply sentences like (15c).

(15)  
   a. The hot liquid is water.
   b. The cold liquid is water.
   c. The hot liquid and the cold liquid (combined) are water.

Uncountables and plurals are thus jointly predicted to have cumulative reference. They are therefore jointly opposed to singulars, which are not predicted to refer cumulatively.

11 Distributive Reference

Uncountables are sometimes said to have distributive reference, a property which is often discussed in conjunction with the minimal parts hypothesis. To be precise about distributive reference, let us say that a subset of a universe is decreasing just in case it contains all the instances of all the kinds it contains. Formally, let $E$ be a universe and let $\leq$ be its relation of instantiation. We will say that some $F \subseteq E$ is decreasing if and only if $l \in E$ and $k \in F$ jointly imply that $l \in F$ whenever $l \leq k$. We may now define the property as follows.

(16) Distributive Reference

An expression will be said to have distributive reference just in case it denotes a decreasing subset of the universe.

Let us consider now Cheng (1973, 287), where uncountable denotations are 'mass objects', a defining property of which is the following: "any part of the whole of the mass object which is $w$ is $w". Or consider more generally McCawley (1981, 439):

Mass terms, whether nouns or adjectives, have the property of being DISTRIBUTIVE: a proposition with a mass predicate implies corresponding propositions about non-empty parts of what is being denoted by the subject; for example, if this soup is warm, then any spoonful of it is warm, and if this ink is red, then any drop of it is red.\textsuperscript{16}

From the standpoint of this study, distributiveness and atomlessness are independent properties of a referent. To see this let us consider a model where an uncountable stem wine denotes the set of all portions of

\textsuperscript{16} "This statement will have to be qualified," McCawley hedges in a footnote, "in that there is, sometimes a lower limit on how small the part may be if the inference is still to be valid. For example, given that this stew is spicy or smelly, it is probably the case that every spoonful of it is spicy or smelly, but not that every cubic millimeter of it is."
wine. This referent will be both atomless and distributive. But let us suppose now that our interpretation has proceeded relative to the model in which all the wine in the universe was contained in a barrel. As shown above, there will be another uncountable stem, call it \textit{wine'}, which denotes the subdomain associated to the wedge shaped portions of wine contained in the barrel. \([\text{wine'}]\) is atomless but not distributive, as the top and bottom halves of wine in the barrel are parts of the wine in the universe even though they are not contained in \([\text{wine'}]\). Conversely, let us consider the domain of any atomistic kind. This set, which is the potential denotation of a countable stem, will be distributive without being atomless.

As far as nominal stems are concerned, they will always have distributive reference when taken in their primary meanings (i.e. when they denote domains in the universe). They will fail to have distributive reference when they are taken in any 'proper' secondary meanings they may have (i.e. when they denote subdomains in the universe which are neither empty nor full). Similar points may be made about plural and uncountable nouns. Singular nouns will also have distributive reference when taken in their primary meanings (every set of atoms of the universe may serve as a trivially distributive referent). But the reference (if any) of dual, trial, and quadral nouns, will never be distributive.

12 Cardinal Adjectives

It will be recalled that every cardinal adjective denotes a function over atomistic subdomains in the universe. But the only atomistic subdomain which an uncountable noun can denote is the empty set. It follows that the only possible denotation for the combination of a cardinal adjective and an uncountable noun will be the empty set—at least if the semantics of this combination is governed by the Functional Principle. Consider for instance the following, where \textit{wine} is taken to be an uncountable noun.

\[(17) \quad \llbracket \text{one wine} \rrbracket = \llbracket \text{one} \rrbracket (\llbracket \text{wine} \rrbracket) = \llbracket \text{one} \rrbracket (\emptyset) = \emptyset\]

Since \textit{one wine} will denote the empty set regardless of the model against which it is interpreted, the phrase is contradictory. The illformedness of this (and any other) combination of \textit{one} and an uncountable noun is thus accounted for.

But consider now cardinal adjectives other than \textit{one}. Recall first that the external syntax of these adjectives requires them to agree in number with their head nouns (recall e.g. \textit{*one scissors}, which is ungrammatical even when we refer to one single artifact). It follows that
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*two wine* will fail syntactically (on account of agreement failure) and that *two wines* will fail semantically (on the same grounds as *one wine*).

But suppose that *wine* were instead taken to be a stem which denoted the set of kinds of wine diagramed in (5). When *wine* is taken in this sense, the phrases *one wine, two wines, three wines* are all well formed and denote as follows.\(^{17}\)

\[
\begin{align*}
\text{(18a)} & \quad \llbracket \text{one wine} \rrbracket = \llbracket \text{one} \rrbracket(\llbracket \text{wine} \rrbracket) = \{x, y, z\} \\
\text{(18b)} & \quad \llbracket \text{two wines} \rrbracket = \llbracket \text{two} \rrbracket(\llbracket \text{wines} \rrbracket) = \{x+y, x+z, y+z\} \\
\text{(18c)} & \quad \llbracket \text{three wines} \rrbracket = \llbracket \text{three} \rrbracket(\llbracket \text{wines} \rrbracket) = \{x+y+z\}
\end{align*}
\]

Interestingly, the phrase *four wines* will be well formed even though it denotes the empty set in the present reinterpretation (there are no four disjoint kinds of wine when we interpret *wine* the way we have). Crucially, however, the phrase will not denote the empty set in *every* admissible model.

Once again, the compatibility between the countable noun *wine* and cardinal modification contrasts with the incompatibility exhibited in this respect by the uncountable noun *furniture*. The phrases *one furniture, two furnitures, three furnitures* are simply ill formed when the head noun is uncountable.

13 The Measure of Kinds

It will be recalled that the multal adjective *many* and the paucal adjective *few* have been interpreted as functions over atomistic subdomains in the universe. But uncountable stems denote atomless subdomains instead. Since the only subdomains which are both atomistic and atomless are empty, the application of \[\llbracket \text{many} \rrbracket\] and \[\llbracket \text{few} \rrbracket\] to the denotations of uncountable nouns will yield only the empty set. It follows from this and the Functional Principle that the combination of our multal and paucal adjectives with uncountables will denote the empty set in every model. The illformedness illustrated in (19) is thus accounted for.

\[
\begin{align*}
\text{(19a)} & \quad ?\text{many wine(s)} \\
\text{(19b)} & \quad ?\text{few wine(s)}
\end{align*}
\]

But consider now the multal adjective *much* and the paucal adjective *little*. As shown by the well known facts in (20)–(21), these adjectives

\(^{17}\) It is worth noting that the countable reinterpretation of *wine* does not redeem *two wine*, adding further support to our view that cardinal adjectives are syntactically required to agree in number with their head nouns.
may combine meaningfully with uncountables but not with countables.  

(20) a. much wine  
    b. little wine  

(21) a. ?much ring(s)  
    b. ?little ring(s)  

To account for these facts we should recall \( \mu(P) \), a function which in effect 'measured' any kind \( k \) in an atomistic subdomain \( P \) in terms of the number of atoms of \( P \) which constituted \( k \). We now wish to be more specific about \( \mu \) so as to have measures for atomless domains as well. To do so we will assume that every subdomain \( P \) of the universe admits of a function \( \mu(P) \) which assigns a positive real number (if not infinity) to each kind \( k \in P \). We will in addition assume that if \( P \) is an atomistic subdomain, then \( (\mu(P))(k) \) is the number of atoms of \( P \) which constitute \( k \), so what we say here will be compatible with the characterization of \( \mu \) in the preceding chapter. We will finally assume that \( \mu(P) \) satisfies the following property. It states that the 'measure' of the mereological sum of two disjoint kinds is equal to the arithmetical sum of their 'measures'.

(22) FINITE ADDITIVITY: If \( x \) and \( y \) are two disjoint kinds of a subdomain \( P \), then \( \mu(P)(x+y) = \mu(P)(x) + \mu(P)(y) \).

The effect of \( \mu(P) \) when \( P \) is atomistic has been illustrated already in Chapter 4. The reader should only note that the function which counts the atoms making up a kind in a given atomistic subdomain satisfies the desired properties (it is finitely additive and its value is a positive integer when it is not infinity).

Let us illustrate then the effect of \( \mu(P) \) when \( P \) is atomless. Suppose once again that all the wine in the universe is collected in one vast barrel. Let us say now that \( P \) is the atomless subdomain generated by the wedge shaped portions of wine contained in the barrel. But suppose that \( \mu(P) \) is the function which assigns to each element of \( P \) its volume. Notice that the volume of each element of \( P \) will be a positive real number (we assume that no portion of wine can have a negative or infinite volume). Notice finally that \( \mu(P) \) is finitely additive (the volume of the mereological sum of two disjoint portions of wine in \( P \) will be equal to the arithmetical sum of their volumes therein).

---

\(^{18}\) The nonpaucal adjective of size *little* may of course combine with countable nouns like *ring(s)*.

\(^{19}\) Notice that we have relied on context to clarify the sense (mereological or arithmetical) in which '+' is taken in the formulation of finite additivity.
Let us observe now that every admissible model may provide a function \( \mu \) such that, for every subdomain \( P \) of its universe there exists a function \( \mu(P) \) which assigns a positive real number (or infinity) to each kind \( k \in P \), is finitely additive, and agrees with the 'height function' of preceding chapters.\(^{20}\) We will say that any such \( \mu(P) \) is a \textit{measure} for \( P \) and claim that \( \mu(P) \) will in fact measure (in the pretheoretical sense) each kind of \( P \) in terms of its instances.

We should note that it can be shown that if \( x \) is a (possibly improper) subkind of \( y \) then, regardless of the subdomain against which they are evaluated, the measure of \( x \) will be (possibly improperly) smaller than the measure of \( y \). Notice finally that since every universe \( E \) is itself a subdomain, \( \mu(E) \) would be a measure for the entire universe. Thus, if \( E \) could be the set of solids in space, \( \mu(E) \) could be the function which assigns, to each solid, its volume. More to the point, if \( E \) is the set of portions of matter in the world, \( \mu(E) \) can be the function which assigns to each portion of matter its mass.\(^{21}\)

14 Multal and Paucal Adjectives

Relative to models which provide a specific function \( \mu \) which satisfies the properties given in the preceding section, the interpretation of the multal adjective \textit{much} and the paucal adjective \textit{little} may now proceed as follows.

(23) \textit{Multal and Paucal Adjectives (Uncountable Version)}

a. The adjective \textit{much} denotes the function which assigns, to each atomless subdomain \( P \), the set \( \{ x \in P : [\mu(P)](x) \text{ is large in } P \} \).

b. The adjective \textit{little} denotes the function which assigns, to each atomless subdomain \( P \), the set \( \{ x \in P : [\mu(P)](x) \text{ is small in } P \} \).

Thus, when presented with an appropriate set of kinds, the multal adjective \textit{much} selects the kinds whose measure in that set is large. The paucal adjective \textit{little}, on the other hand, selects the kinds in a set which are small in that set. Notice once again that what counts as

\(^{20}\) Notice that no model will fail to be admissible for want of \( \mu \), since every model may have the following trivial \( \mu \). For all \( k \in P \), let \( [\mu(P)](k) \) be either the number of atoms of \( P \) which constitute \( k \) (if \( P \) is atomistic) or infinity (if \( P \) is not atomistic).

\(^{21}\) The notion of measure adopted for this study is the transposition of the boolean notion of (strictly positive) measure into a mereological setting. See for instance Sikorski (1969; 11, 73).
large or small will be left open, but must in any event depend on the subdomain in question and on the measure provided by the model (and all the points made about the contextualization, fuzziness, and ambiguity of multal and paucal adjectives in the preceding chapters carry over to this one).

As the reader will readily verify, the adjectives interpreted in (23) differ from their countable counterparts solely with respect to the mereological structure of the sets from which they make their selections. Many and few may select only from atomistic subdomains, whereas much and little may select only from atomless subdomains. Such a difference is welcome, as it seems to match intuition closely.

To illustrate the force of (23), let us consider an uncountable noun wine which denotes an atomless subdomain $W$. Given (23) and the Functional Principle, we have the interpretations in (24) for the expressions in (20).

\begin{align*}
(24) \quad a. \quad \llbracket \text{much wine} \rrbracket &= \llbracket \text{much} \rrbracket (\llbracket \text{wine} \rrbracket) = \{ x \in W : [\mu(W)](x) \text{ is large in } W \} \\
&= \{ x \in W : [\mu(W)](x) \text{ is large in } W \} \\
&= \{ x \in W : [\mu(W)](x) \text{ is small in } W \} \\
&= \{ x \in W : [\mu(W)](x) \text{ is small in } W \}
\end{align*}

But the application of the functions in (23) to countable nouns like rings will be undefined. The contrast between (20) and (21) thus follows, as desired, from our proposals.

Now, it might be objected that the use of the adjectives much and little does not presuppose the knowledge of a numerical function but only that of a ‘ranking’, possibly continuous, of amounts of stuff. But the requirement of finite additivity presupposes that of a numerical function (and it is doubtful that the notion of measure can survive any weakening of this requirement). Moreover, the ordinary use of cardinal adjectives described in the preceding chapter seems to involve knowledge of integers (and hence of a numerical function).

In any event, the usual points about the secondarily countable senses of wine can be made in the present context. For, if there is a noun wine which denotes the atomless set of portions of wine, then there will also be a noun wine which denotes an atomistic subdomain like the one in (5). With such a noun, only many and few, not much or little, may be used. And the familiar points concerning the linguistic irrelevance of the seemingly minimal parts of [furniture] can also be made in the present setting.²²

²² See Section 16 below for further discussion of constructions with multal and paucal adjectives.
15 Quantifiers

We turn next to quantifiers, whose interaction with uncountables may be illustrated by the contrasts in (25)-(28). Again, the subjects of the questioned versions are taken in their uncountable rather than varietal senses.

(25) a. ?Every wine is red.
    b. All wine is red.

(26) a. ?A wine is red.
    b. Some wine is red.

(27) No wine is red.

(28) a. ?Both wines are red.
    b. ?Neither wine is red.

As the reader will be able to verify, all these contrasts follow given the uncountability of *wine* and conditions (25), (31), (38), (45) which have been assigned, respectively, to the universal, the existential, the nonexistential, and the dual quantifiers, in Chapter 4. Thus, in all the questioned versions, a function is applied to a set which is pairwise disjoint (or the closure thereof) only if empty. These sentences would therefore be interpretable only if they contain a necessarily empty (hence contradictory) noun.

It will be noticed that conditions (25), (31), (38) of Chapter 4 (and the Functional Principle) will allow us to interpret sentences with countables and with uncountables in strictly parallel fashion:

(29) a. All rings are expensive.
    b. $[[\text{rings}]] \subseteq [[\text{expensive}]]$

(29') a. All gold is expensive.
    b. $[[\text{gold}]] \subseteq [[\text{expensive}]]$

(30) a. Some rings are expensive.
    b. $[[\text{rings}]] \cap [[\text{expensive}]] \neq \emptyset$

(30') a. Some gold is expensive.
    b. $[[\text{gold}]] \cap [[\text{expensive}]] \neq \emptyset$

(31) a. No rings are expensive.
    b. $[[\text{rings}]] \cap [[\text{expensive}]] = \emptyset$

(31') a. No gold is expensive.
    b. $[[\text{gold}]] \cap [[\text{expensive}]] = \emptyset$
It will also be noticed that we can predict that uncountables may combine with predicates like *looks alike* which, we have assumed, select the nonatoms of a subdomain which look alike:

(32) All gold looks alike.

Here the subdomain to which \([looks alike]\) applies is atomless: it is the one denoted by (the stem) *gold.*

It should not escape the reader that our account of (32) was made possible, in part, by our previous assumption that the singular inflection on *looks alike* is a syntactic reflex of no semantic consequence. The said assumption can therefore be justified on independent grounds. As to the by now familiar points about the secondarily countable senses of *wine* and the seemingly minimal parts of *furniture,* they can also be made with the quantifiers discussed in this section.

But let us suppose now that all the ink of the world is found in two distinct containers. Suppose further that all the ink in one container is red, while all the ink in the other is black. Is (33) true under such states of affairs?

(33) All ink is either red or black.

Actually, the answer is 'no', for there are portions of ink which are neither red nor black. Consider for instance the portion of ink which is constituted by the top half of one container and the bottom half of the other. That portion of ink, albeit disconnected, will be neither red nor black. Naturally, (34) should also be false.

(34) All portions of ink are either red or black.

Operating under the assumption that sentences like (33) are *true* and sentences like (34) are *false,* Roeper (1983) concluded that an uncountable noun could not denote a set (of portions), but rather an individual (portion). But such an assumption seems to us unwarranted: both (33) and (34) should be false. We should point out, however, that in the process of developing his own analysis, Roeper proposes an important theory of complex predicates to which we shall return below.

---

23 But uncountables will not combine with all 'collective' predicates. The verb *meet,* for instance, takes plural, not uncountable subjects:

He planted some trees and watched them meet.

?He planted some grass and watched it meet.

See here Gruber (1967, 236), who moreover claims that the converse selectional restrictions will not be attested, and thus proposes a system of semantic representation which exploits this asymmetry.
16 Multal, Paucal, and Cardinal Noun Phrases

Expressions like *many rings, few men, much wine,* and *little gold* often seem to function not merely as modified common nouns, but rather as complete noun phrases. When they do, each should denote what we have assumed every noun phrase will denote: a family of subsets of the universe. To be more specific, we will now propose that they denote as indicated in (35) and (36), where *N* is a noun (or nominal) which is countable in (35) and uncountable in (36).

(35) a. \[[many \text{N}] = \{Q \subseteq E: [\text{many}][([N])] \cap Q \neq \emptyset\}\]
   b. \[[\text{few}\ \text{N}] = \{Q \subseteq E: [\text{not}\ \text{few}][([N])] \cap Q = \emptyset\}\]

(36) a. \[[\text{much}\ \text{N}] = \{Q \subseteq E: [\text{much}][([N])] \cap Q \neq \emptyset\}\]
   b. \[[\text{little}\ \text{N}] = \{Q \subseteq E: [\text{not}\ \text{little}][([N])] \cap Q = \emptyset\}\]

It will be noticed that when the expressions interpreted in (35a) and (36a) are used as noun phrases, they will denote the families of conjoints of the sets they denote when used as modified nouns. Less transparently, when the expressions interpreted in (35b) and (36b) are used as noun phrases, they will denote the families of disjoints, not of the sets they denote when used as modified nouns, but rather of the *complements* thereof (in \[[N]\]).

To justify these proposals, let us observe that if we combine these interpretations with those of the multal and paucal adjectives of this and the preceding chapter, then we obtain the interpretations in (35') and (36'). To ease readability, we use *P* as short for \[[\text{N}]\].

(35') a. \[[\text{many}\ \text{N}] = \{Q \subseteq E: (x \in P: [\mu(P)](x) \text{ is large in } P) \cap Q \neq \emptyset\}\]
   b. \[[\text{few}\ \text{N}] = \{Q \subseteq E: (x \in P: [\mu(P)](x) \text{ is not small in } P) \cap Q = \emptyset\}\]

(36') a. \[[\text{much}\ \text{N}] = \{Q \subseteq E: (x \in P: [\mu(P)](x) \text{ is large in } P) \cap Q \neq \emptyset\}\]
   b. \[[\text{little}\ \text{N}] = \{Q \subseteq E: (x \in P: [\mu(P)](x) \text{ is not small in } P) \cap Q = \emptyset\}\]

These interpretations can be illustrated by considering (37) and (38), where *M* is short for \[[\text{men}]\]. Entirely analogous equivalences are of course available for the uncountable cases.

(37) \[[\text{many men came}] \leftrightarrow [\text{came}] \in [\text{many men}]\]
\[\leftrightarrow [\text{came}] \in \{Q \subseteq E: [[\text{many}][([\text{men}])] \cap Q \neq \emptyset\}\]
\[\leftrightarrow [\text{came}] \cap [[\text{many}][([\text{men}])] \neq \emptyset\]

---

24 See Chapter 4, Section 11.
\[ \leftrightarrow \llbracket \text{came} \rrbracket \cap \{ x \in M : [\mu(M)](x) \text{ is large in } M \} \neq \emptyset \]

\(\llbracket \text{few men came} \rrbracket \leftrightarrow \llbracket \text{came} \rrbracket \in \llbracket \text{few men} \rrbracket \]
\[ \leftrightarrow \llbracket \text{came} \rrbracket \in \{ Q \subseteq E : \llbracket \text{not few} \rrbracket(\llbracket \text{men} \rrbracket) \cap Q = \emptyset \} \]
\[ \leftrightarrow \llbracket \text{came} \rrbracket \cap \llbracket \text{not few} \rrbracket(\llbracket \text{men} \rrbracket) = \emptyset \]
\[ \leftrightarrow \llbracket \text{came} \rrbracket \cap \{ x \in M : [\mu(M)](x) \text{ is not small in } M \} = \emptyset \]

So many men came asserts that the set of individuals that came overlaps with the set of 'men in large numbers', whereas few men came asserts that the set of individuals that came is disjoint from the set of 'men in not small numbers'.

Now, it might be thought that few men came should more transparently assert that the set of individuals that came overlaps with the set of 'men in small numbers'. This, however, cannot be so. For, suppose that many men indeed came. Suppose further that if many men came, then a small number of those men came. It follows that few men came would be true if many men came—hardly a welcome result.\(^{25}\) Nothing of the sort is true under the proposed interpretation. If many men came, then the set of individuals that came overlaps with the set of 'men in not small numbers'—and few men came would thus be false as desired. Similar points can be made about the uncountable little in sentences like little water boiled.

Moreover, suppose there is a meeting which no man attended. Suppose further that this was not for want of men, as there were many men elsewhere. Since I saw almost all of the men outside the meeting, I can assert (39) without contradiction.

\[ \llbracket \text{attended the meeting} \rrbracket \land \{ x \in M : [\mu(M)](x) \text{ is not small in } M \} = \emptyset \]

which shows that the first conjunct is compatible with the possibility that no men attended the meeting. This is a fact about few that the proposed account can handle. For, to say that no men attended the meeting is to say that \(\llbracket \text{attended the meeting} \rrbracket \) and \(\llbracket \text{men} \rrbracket \) are disjoint. But if they are, then \(\llbracket \text{attended the meeting} \rrbracket \) and the less inclusive set of men in nonsmall numbers would have to be disjoint as well:

\[ \llbracket \text{attended the meeting} \rrbracket \cap \{ x \in M : [\mu(M)](x) \text{ is not small in } M \} = \emptyset \]

\(^{25}\)I am indebted to Jan Tore Lønning for making me aware of this point.
But (40) is, we claim, the assertion made by the first conjunct in (39). It thus follows from our claims that *few men attended the meeting* may be true when no men attended the meeting. In fact, it *must* be true then.

Interestingly, the fact borne out by (39) cannot be handled if *few men attended the meeting* were to make the more transparent assertion considered above—namely that \([\text{attended the meeting}]\) and \([\text{few men}]\) overlap. For again, to say that no men attended the meeting is to say that \([\text{attended the meeting}]\) and the set of men are disjoint. But if they are, then \([\text{attended the meeting}]\) and the less inclusive set of men in small numbers would have to be disjoint as well. Hence \([\text{attended the meeting}]\) and \([\text{few men}]\) will not overlap, and the first conjunct in (40) would have to be false when no men attended the meeting. (40) would thus be incorrectly predicted to be illformed on grounds of contradiction.

Let us assume then that the interpretations in (35) and (36) are on the right track. The question then arises as to how these interpretations are attained. One attractive possibility is to propose phonetically null determiners which construct the desired families by applying to the denotations of *many rings* or *little gold* taken as modified common nouns. It is not difficult to see what these null determiners would have to denote. Moreover, the null determiner purportedly involved in the multal cases would be synonymous to one which is lexically realized elsewhere—namely the existential determiner *some* with which we are by now familiar.

Parallel situations arise for countable nouns modified by cardinal adjectives. Thus, expressions like *one ring* and *two men* often seem to function not merely as modified common nouns, but rather as complete noun phrases. When taken in this way, they can be taken to denote as follows, where \(R\) is an atomistic subdomain of rings and \(M\) is an atomistic subdomain of men.

\[
(41) \begin{align*}
\text{a. } [\text{one ring}] &= \{Q \subseteq E: [\text{one}](R) \cap Q \neq \emptyset\} \\
\text{b. } [\text{two men}] &= \{Q \subseteq E: [\text{two}](M) \cap Q \neq \emptyset\}
\end{align*}
\]

The general case is given in (42), where \(n\) is an arbitrary cardinal adjective and \(P\) is the atomistic subdomain denoted by the stem of \(N\).

\[
(42) [\text{n } N] = \{Q \subseteq E: [\text{n}](P) \cap Q \neq \emptyset\}
\]

So when (42) is taken in conjunction with the interpretations of cardinal adjectives presented in Chapter 4, the interpretations in (41) become

\[
(41') \begin{align*}
\text{a. } [\text{one ring}] &= \{Q \subseteq E: [x \in R: [\mu(R)](x) = 1] \cap Q \neq \emptyset\} \\
\text{b. } [\text{two men}] &= \{Q \subseteq E: [x \in M: [\mu(M)](x) = 2] \cap Q \neq \emptyset\}
\end{align*}
\]

This means that *two men* will denote the family of sets which overlap with the set of "men in twos". Equivalents of existential
determiners can thus be discerned in both cardinal and multal noun phrases.

Evidence for the analysis in (42) is not difficult to provide. Suppose three men came. But if three men came, then so did two of those men. Hence the set of individuals that came contains a sum of two men. This means that the set of individuals that came overlaps with the set of 'men in twos'. But this suffices to make the sentence two men came true—at least if (42) is adopted. The interpretation in (42) thus allows us to derive the correct prediction that two men came will be true if three men came. Interestingly, however, the prediction can be derived even though the modified noun two men denotes the set of exactly two men!

17 The Quasiuniversal Determiner

We now wish to interpret the 'quasiuniversal' determiner most as indicated in (43). It will be noticed that the semantic effect of this determiner is the construction of families of sets which are akin to the ones in (35'a) and (36'a). More precisely, \([most]\) will construct families which will differ from the said families only with respect to the mereological structure of \(P\) and the demands placed on \([\mu(P)](x)\).

(43) **The Quasiuniversal Determiner**

The determiner most denotes the function which assigns, to each nonempty subdomain \(P\), the family \(\{Q \subseteq E : \{x \in P : [\mu(P)](x)\text{ is very large in } P\} \cap Q \neq \emptyset\}\).

Our determiner will thus construct, from an appropriate \(P \subseteq E\), the family of sets \(Q \subseteq E\) which contain a member of \(P\) whose measure in \(P\) is 'very large'. The interpretation proposed in (43) can be illustrated with the denotations of most rings and most wine (relative to universes which contain wine and rings):

(44) \([most \text{ rings}] = [most][\text{[rings]}]\)

\[= \{Q \subseteq E : \{x \in R^* : [\mu(R^*)](x)\text{ is very large in } R^*\} \cap Q \neq \emptyset\}\]

(45) \([most \text{ wine}] = [most][\text{[wine]}] = \{x \in W : [\mu(W)](x)\text{ is very large in } W\}

\[\{Q \subseteq E : \{x \in W : [\mu(W)](x)\text{ is very large in } W\} \cap Q \neq \emptyset\}\]

The application of \([most]\) to (nonvacuous) plural and uncountable nouns is thus defined. Incidentally, the application of most to a singular noun denotation is defined whenever the latter is a singleton (every singleton is an atomistic subdomain). We must therefore regard the
illformedness of the expression in (46) as one which is syntactic rather than semantic in nature.

(46) *most ring

Consider next the stipulated nonemptiness of the subdomains to which \([\textit{most}]\) applies. As far as we can determine, this stipulation is forced on us by the facts of language: the sentences in (47) are decidedly odd.

(47) a. ?Most unicorns are white—although there are no unicorns.
    b. ?Most phlogiston is volatile—although there is no phlogiston.

It should be noted that (43) requires \([\mu(P)](x)\) to be very large relative to \(P\); it does not require it to be very large \textit{simpliciter}. It should also be noted that however (in)accurate this requirement may turn out to be, it will account for the intuition that \textit{most} makes a more stringent contribution to the truth conditions of a sentence than \textit{many}. In addition, it will ensure that \textit{most} will inherit from \textit{many} the vagueness and context-dependence inhering in 'large' (see Chapter 3, Section 10, and the references cited therein). We observe that none of this would necessarily follow if we were to adopt the usual analysis and replace 'very large' in (43) by 'more than half' (and then retract from the increment in precision brought about by the latter).

We conclude this section with the observation that the semantics of \textit{most} advanced herein can provide for an unproblematic account of the sentence in (48) relative to a universe in which there is far more unmined gold than mined gold (and all the unmined gold is of a piece while all the mined gold is scattered in a myriad pieces of jewelry).

(48) Most gold is still under ground.

Relative to such a universe, the sentence would simply be true (the 'individuation' induced by spatial scattering being of no consequence to the truth value of the sentence). As we see them, sentences like (48), originally proposed in Parsons (1970), are interesting because they show

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26 As an anonymous reviewer has pointed out, \textit{most} does not seem to be as dependent on context as \textit{many}. If small numbers like 16 can be contextually large with \textit{many}, a similar effect should be possible with moderate amounts like 20%, which is contrary to fact:

Many houses are insured in San Francisco: 15,000.
Many houses burned down in San Francisco last year: 16.
Most houses are insured in Dodge City: 60%.
?Most houses burned down in Dodge City: 20%.
that [[most]] is sensitive to size rather than to number of portions (and that portions need not be isolated from each other in space). They thus contribute to the case for imposing a structure on the domain of individuals provided by an admissible model.

18 The Definite Article

The semantics of uncountability presented in this chapter is fully compatible with the semantics of the definite determiner we have proposed in the preceding chapters. Thus, if \( W \) stands for the nonempty atomless subdomain denoted by some uncountable noun *wine*, and if \( w \) stands for the sum of all portions of wine, then *the wine* would denote as indicated in (49).

\[
(49) \quad [[\text{the wine}]] = [[\text{the}]]([[\text{wine}]]) = \Gamma(W) = w
\]

Naturally, if *wine* denotes the empty set, then the denotation of *the wine* would be undefined. For, as interpreted above, the definite article denotes a function whose value is undefined when it applies to empty subdomains. It will therefore be undefined when they apply to empty atomless subdomains.

It is important to remember that the set denoted by an uncountable noun is infinite if it is not empty. Interestingly, however, the denotation of *the wine* is always defined as long as the noun is not semantically vacuous. It follows from our proposals (correctly and with a vengeance) that the definite article does not always have a presupposition of uniqueness. In fact, the proposals advanced in this study will predict that the definite article has a presupposition of uniqueness only when it combines with (semantically) singular nouns; it will lack the said presupposition when it combines with plurals and uncountables.

To see how this prediction is derived, notice that our proposals imply that a definite description will denote properly just in case its predicate denotes a set with a greatest element. But such an element will be secured if the set is complete and nonempty. As to completeness, it will be secured if the set is a subdomain in the universe. But that is what each plural and uncountable noun has been constrained to denote. A singular noun denotation, on the other hand,

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27 See also McCawley (1981, 436).
28 Notice that definite determiners have been assigned here their 'morphological denotations'. This is done only to simplify matters. All the points we will make will hold, *mutatis mutandis*, if determiners are assigned their 'syntactic denotations' (see the sections on syntactic denotations).
will contain a greatest element only when it is a singleton, as no other pairwise disjoint set can have a greatest element.

The problem raised by uncountables for the Russellian presupposition of uniqueness was noted in Cartwright (1965, 481). Our solution, which is essentially the one proposed in Sharvy (1980), has been to regard the definite article as the \textit{gamma} rather than the \textit{iota} operator. Paraphrasing Sharvy (1980, 623), the primary use of the definite descriptions is not to refer to unique elements. Rather, it is to refer to greatest elements. The implication of uniqueness in the case of singulars is but a side effect.\(^{29}\)

It should be clear that the proposed solution to the problem noted by Cartwright is an improvement over the one suggested in Montague (1973a, 290f), where the gold in Smith's ring is analyzed by assuming \textit{ad hoc} that the nominal \textit{gold in Smith's ring} denotes not the set of portions of gold in Smith's ring, but only the set of \textit{maximal} portions of gold in Smith's ring—a singleton, as desired. Although the set of maximal portions of gold may be the denotation of our nominal, we do not have to stipulate that it must be.

Be that as it may, the proposals concerning the semantics of the definite article and the semantics of uncountables are corroborated by the validity of sentences like (50), which we take as asserting that an atomless kind (here the water in the universe) is contained in its own domain (here the set of portions of water in the universe).

\(50\) \text{The water (there is) is water.}

The situation is entirely analogous to the one which holds for our proposals concerning the definite article and the semantics of plurality. Consider sentences like the one in (51), which we take as asserting that an atomistic kind (here mankind) is contained in its own domain (here the set of subkinds of mankind).

\(51\) \text{The men in the universe are men in the universe.}

And the same holds for the definite article and our proposals concerning the semantics of singularity. For consider the validity of (52), which asserts that the whale is a whale. Notice that (52) could be either about a particular whale or about the entire kind of whales.

\(52\) \text{The whale is a whale.}

Moreover, let us consider uncountable generic statements like (53a) and their universal counterparts (53b). It will be noticed that the former

\(^{29}\) The proposals of Sharvy (1980) have been discussed in more detail in the section on the definite article of Chapter 3.
do not imply the latter. As we see it, (53a) simply states that the maximal portion of water is poisoned without implying that every part of that portion is. (53b) normally states, on the other hand, that every portion of water—the maximal portion included—is poisoned.

(53) a. The water is poisoned.
    b. All water is poisoned.

But this follows directly from our proposals, since the definite article denotes the greatest function while the universal quantifier denotes the superset function. Our account of the defeasibility of uncountable generic statements is thus of a piece with that of countable generic statements.

19 On the Protean Nature of Mass Nouns

We would next like to turn to the well known ambivalence claimed for uncountables in Quine (1960, §20). According to this philosopher, uncountable nouns (or 'mass terms') can occur either before the copula (as 'subjects') or after the copula (as 'predicates'). In the first case they denote sets (they are thus 'general terms'), while in the second they denote individuals (they are therefore 'singular terms'):

Examples showing mass terms after 'is' are 'That puddle is water', 'The white part is sugar', 'The rest of the cargo is furniture'. [...] We can view the mass terms in these contexts as general terms, reading 'is water', 'is sugar', 'is furniture', in effect as 'is a bit of water', 'is a bit of sugar', 'is a batch of furniture'. In general a mass term in predicative position may viewed as a general term which is true of each portion of the stuff in question [...] In 'Water is a fluid', on the other hand, and 'Water is fluid', and 'Water flows', the mass term is much on a par with the singular term of 'Mama is big' or 'Agnes is a lamb' [...] We shall do best to acquiesce in a certain protean character on the part of mass terms, treating them as singular in the subject and general in the predicate (cf. Quine 1960, 97f).

As we see it, a mass noun qua noun always denotes an atomless subdomain. It is, therefore, what Quine would call a general term. But, when an uncountable noun occurs in subject position it seems to us that it does so in conjunction with a phonetically null definite article which would select the greatest element of the set denoted by the mass noun.

To support this view let us notice first that such definite articles are in fact audible in languages other than English (cf. French L'eau est un fluide or German Das Wasser ist eine Flüssigkeit). Notice also that positing phonetically null definite article is not without precedent in English. The proposal is akin to one standardly invoked for frequently occurring nouns like man (cf. Man is mortal) or woman (cf. Woman is fickle). What is more, the inaudibility of such definite articles has been
tentatively explained by Jespersen (1954, II, §5.411) as a remnant from the time when articles were not in general use. Finally, notice that phonetically realized articles are not an option in this construction (cf. *The water is a fluid)—as one might expect of subjects with two articles.\(^{30}\)

In any event, it follows that we can provide a unified account of the validity of (50) above and (50') below.

(50') Water is water.

Moreover, we can account for the fact that the subject of (54) can tolerate proper kind predication. The said subject denotes the greatest portion of water—a kind constructed by the application of a phonetically null definite article to the set denoted by the noun water.\(^{31}\)

(54) Water is widespread / common / fast disappearing / often impure / that kind of liquid.

But the protean nature of mass terms has been called to question by 'the puddle puzzle', which consists in accounting for the validity of the inference in (55).

(55) This puddle is water

\[ \text{Water is wet} \]

This puddle is wet

For indeed, if water denotes a set of individuals in the first premise and an individual in the second, then the assertions involved in (55) are as indicated in (55'), which does not follow a valid form of inference.

(55') \[ \{x: x \text{ is this puddle}\} \subseteq \{x: x \text{ is a portion of water}\} \]
\[ \{x: x \text{ is the greatest portion of water}\} \subseteq \{x: x \text{ is wet}\} \]
\[ \{x: x \text{ is this puddle}\} \subseteq \{x: x \text{ is wet}\} \]

It might indeed seem that “one must treat mass terms as being either predicates or individual constants but not both, on pain of failing to account for the logical relations binding different sentential occurrences together” (cf. Burge 1972, 267).

It should be clear that our refinement of Quine's view of the protean nature of mass terms inherits the puddle puzzle. Our solution to the puzzle is to propose that wet is an expression with distributive

\(^{30}\) Observe that ours is only a refinement of, not an alternative to, the view expressed in Quine (1960). For when the expression man constitutes the subject of a sentence, it will denote a set qua noun but an individual qua noun phrase.

\(^{31}\) These sentences should be compared with The owl is widespread / common / fast disappearing / often intelligent / that kind of bird discussed in the section on the definite article in Chapter 3.
reference: any portion of a wet individual is itself wet (notice that wet is taken here as describing every portion of water rather than only its surface). Given this proposal, the second premise implies that all water is wet, i.e. that \( \{x: x \text{ is a portion of water}\} \subseteq \{x: x \text{ is wet}\} \). By substituting this for the second premise, we produce a valid form of inference:

\[
(55') \quad \{x: x \text{ is this puddle}\} \subseteq \{x: x \text{ is a portion of water}\}
\]
\[
\{x: x \text{ is a portion of water}\} \subseteq \{x: x \text{ is wet}\}
\]
\[
\{x: x \text{ is this puddle}\} \subseteq \{x: x \text{ is wet}\}
\]

Evidence that the validity of the argument depends on the distributive reference of wet is provided by the fact that the inference is invalid if wet is replaced by a nondistributive predicate like widespread.\(^{32}\)

\[
(56) \quad \text{This puddle is water}
\]

\[
\text{Water is widespread}
\]

\[
\text{This puddle is widespread}
\]

A similar point can be made if wet is replaced by clear. For, given that this puddle is water and given that water, viewed as a kind, is clear, we cannot conclude that this puddle is clear.\(^{33}\)

20 Uncountability and Intensionality

We thus claim that any subject which is constituted by an uncountable noun and a (possibly null) definite article will denote a mereological sum. But such claims have been criticized in the literature on three different grounds (cf. Pelletier and Schubert 1989, 358).\(^{34}\) Consider first a universe of discourse in which the sum of all portions of furniture coincides with the sum of all portions of wood. Now, it is stated that since identity of wholes is defined as containing all the same parts, we would have

\[
(Ax)(x < \text{Wood} \iff x < \text{Furniture})
\]

But this is false, even in the imagined circumstances. The leg of this chair is wood but is not furniture. The reason for this is that they have different minimal parts \[\ldots\] Of course if one denies that there are minimal parts of any mass noun, this argument will not seem to carry much force \[\ldots\] ; but we feel that it should be bothersome. Any proposal which flies in the face of the "primary semantic data" (here: claiming that the leg of the chair is furniture) like this should be...

\(^{32}\) The conclusion of this invalid inference is in addition semantically illformed if this puddle is taken to denote (the singleton constituted by) an atom of the universe. Recall the preceding discussion of sentences like This ring is gold.

\(^{33}\) I owe this example to Manfred Krifka.

\(^{34}\) See also ter Meulen (1981).
adopted only after deep and careful analyses of the alternatives show them to be even more wanting (cf. Pelletier and Schubert 1989, 358).

As has been pointed out, we in fact deny that there are minimal parts of any mass noun denotation. But the present criticism loses its force for a stronger reason, as it involves a *non sequitur*. Quite simply, the sum of \([\text{furniture}]\) and the sum of \([\text{wood}]\) may coincide without the minimal parts of \([\text{furniture}]\) and \([\text{wood}]\) themselves coinciding. Suffice it to recall that the sum of all the (atomless) portions of wine may in fact coincide with the sum of portions in the (atomistic) mereology diagramed in (5) above. Yet, this does not mean that the two sets of portions of wine have the same minimal parts. In fact, one set has minimal parts whereas the other does not!

But the intent of the preceding criticism may be different. Consider a state of affairs in which all furniture is made of wood and all wood has been made into furniture. Here the sum of all portions of wood and the sum of all portions of furniture would coincide. Yet they should not be the same kind (furniture is made by carpenters whereas wood is not). It would seem to follow that the subjects *furniture* and *wood* cannot be taken to denote simple mereological sums in such states of affairs.\(^{35}\)

There is, however, another alternative. Suppose that the nouns *furniture* and *wood* denote different sets which may occupy the same space at the same time. Such a supposition is not without precedent. We should recall the case of (the set constituted by) my ring and the set of portions of gold making it up—as well as the case of the hammock and the rope used to weave it, and the case of the snow and the water making it up. Let us suppose, then, that *furniture* and *wood* denote different sets which may indeed occupy the same space at the same time. It follows that their sums would represent different kinds even in a state of affairs in which all furniture is made of wood and all wood has been turned to furniture. Now \([\text{furniture}]\) can be a subset of the set of artificial entities whereas \([\text{wood}]\) can be a subset of the set of natural entities (and the sets of natural and artificial entities can still be disjoint).

A second criticism of the idea that mass nouns *qua* subjects denote mereological sums runs as follows:

If, for instance, one takes a subject occurrence of *water* as denoting all the water in the world, there will be a "paraphrase" problem, since presumably *all the water in the world* also denotes this entity. But *All the water in the world weighs billions of tons* is intelligible and true, yet *Water weighs billions of tons* seems semantically odd (cf. Pelletier and Schubert 1989, 358).

\(^{35}\) I owe this criticism to Manfred Krifka (personal communication).
But why should it be presumed that *water* (in its subject occurrence) and *all the water in the world* denote the same entity? The contrast cited by these authors provides *prima facie* evidence against the presumed synonymy. And strong evidence against the corresponding identification in the countable domain is provided by the contrasts of Ladusaw (57), Partee (58), and Dowty (59), gathered in Roberts (1986, 211).

[(57) a. The students voted to accept the proposals.  
   b. All the students voted to accept the proposals.](58) a. The trees are denser in the middle of the forest.  
   b. ?All the trees are denser in the middle of the forest.  

(59) a. The students are numerous.  
   b. ?All the students are numerous.  

As we see it, *all the water in the world* is short for *all of the water in the world*, which may well denote as follows:

[(60) all of the water in the world] = [[all]([of])([the])([water in the world])])  
   = [[all]([of])([the](W))]  
   = [[all]([of])(I(W))]  
   = [[all](E | I(W))]  
   = \{P \subseteq E: E \subseteq I(W) \subseteq P\}]

and if the set \( W \) denoted by *water in the world* is a domain (rather than a proper subdomain) in the universe, then

\[ (P \subseteq E: W \subseteq P) = [[all \, water \, in \, the \, world]] \]

The only aspect of this interpretation which has not been discussed before pertains to the semantics of the partitive preposition *of*. As the reader will have noticed, we have interpreted this preposition as a function which assigns, to each kind, its domain. Here it assigned \( E \mid I(W) \) to \( I(W) \).36

The denotation of *all the water in the world* thus contrasts with that of *water (qua subject)* as (60) differs from (61). Notice that (61) provides an interpretation of *water* as a family of sets. This has been done in deference to the views expressed in the sections on 'syntactic

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36 If *the* is assigned its proper syntactic denotation, then the preposition would have to be interpreted as a function which assigns, to each individual sublimation, the domain of that individual. Notice that this does not affect the denotation of *all the water in the world*; it changes only the way in which this denotation is computed.
denotations'. If the analysis of noun phrases as families of sets were abandoned, the semantic gap between our expressions would only widen, as the relevant occurrence of *water* would then denote the individual \( \Gamma(W) \).

(61) \[ \{ P \subseteq E : \Gamma(W) \in P \} \]

Notice that the proposed difference between the denotations in (60) and (61) allows for an explanation of the contrast mentioned in the passage cited above. When *water* denotes (the sublimation of) a kind, it can only tolerate the predication of its 'specific weight', not its 'total weight'. Conversely, since *all the water in the world* denotes (the sublimation of) the domain of a kind, then it can tolerate only the predication of its 'total weight', not that of its 'specific weight'. The contrasts in the passage cited should thus be complemented with the ones in (62).

(62) a. Water is heavy.
    b. ?All the water in the world is heavy.

A third criticism of the idea that subject occurrences of mass nouns denote mereological sums proceeds as follows. Consider a universe which contains, say, neither phlogiston nor ambrosia. In such universes the kind phlogiston and the kind ambrosia would coincide (or so the argument goes), as they would both be "the empty totality, and have the same parts, namely the empty part."37

As it turns out, however, the postulates of mereology imply that there can be no 'empty totality'. The 'sum of all elements of the empty set' is not a notion which mereological theory can define. This means that there simply cannot be a kind without instances. Thus, far from being valid, the sentence in (63) is flat out uninterpretable in models which contain no phlogiston or ambrosia. In such models, \([ (the) \text{phlogiston} ] = [ (the) \text{ambrosia} ] = \Gamma(\emptyset)\), and \( \Gamma(\emptyset) \) is undefined.

(63) Phlogiston is ambrosia.

The problem, if there is one, arises only in the case of nouns, adjectives, or verbs which denote the empty set. But this, of course, is independent of the semantics of countability. Incidentally, it is interesting to point out that (63) is not semantically ill formed. This sentence will be interpretable (and maybe even true) in models which contain both phlogiston and ambrosia.

Taken constructively, the three criticisms discussed above have been said to establish that subject occurrences of uncountables denote

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37 See Pelletier and Schubert (1989, 358), where 'element 130' and 'element 131' are used instead of phlogiston and ambrosia.
intensional entities—a proposal first advanced in Montague (1973a, 289f). Although we do not dispute that these expressions have intensional entities as their meanings, we believe that they have extensional entities as their referents (see Chapter 2). In this, countables and uncountables go hand in hand. As argued in Bunt (1985, 37f), linguistic phenomena that require an intensional framework occur with mass terms in essentially the same way as with count terms (see also Link 1983, 306).

21 Complex Predicates

Let $E$ be once again the set of individuals provided by a model. Let $P(E)$ be the set of all subsets of $E$. Let $\leq_{P(E)}$ be the relation of set theoretic inclusion over $P(E)$. It is easy to show that $P(E)$ and $\leq_{P(E)}$ jointly constitute a boolean algebra. Now, we have assumed in Chapter 1 that predicates select their denotations from among the elements of this algebra (or rather, from the characteristic functions of these elements). Let us suppose now that $\neg$, $\or$, and $\and$ may respectively denote the operations of complementation, join, and meet of this algebra. These operations are, of course, set theoretic complementation, set theoretic union, and set theoretic intersection, respectively. If $[[\text{big}]]$ and $[[\text{tall}]]$ are elements of this algebra, we may then interpret complex predicates as follows.

(64) 
$[[\neg \text{big}]] = [[\exists x \in E: \neg x \in [[\text{big}]]]]$
$[[\text{big or tall}]] = [[\exists x \in E: x \in [[\text{big}]] \or x \in [[\text{tall}]]]]$
$[[\text{big and tall}]] = [[\exists x \in E: x \in [[\text{big}]] \and x \in [[\text{tall}]]]]$

And, as is widely known, it is possible to shift the complexity of these predicates to the sentences which contain them salva veritate:

(65) 
a. John is not big.
   b. It is not the case that John is big.

(66) 
a. John is big or tall.
   b. John is big or John is tall.

(67) 
a. John is big and tall.
   b. John is big and John is tall.

But let us say now that $E$ is the set of individuals provided by an admissible model. This means that $E$ is a universe of discourse, and that it therefore constitutes a mereology when taken in conjunction with the
relation of instantiation provided by the model. Let \( J(E) \) be the set of all the domains (or principal ideals) of \( E \). More formally,

\[
(68) \quad J(E) = \{ E \mid k: k \in E \}
\]

Let us define now a binary relation, call it \( \leq_{J(E)} \), over \( J(E) \) as indicated in (69). Notice that to avoid confusion we will let \( \leq_E \) stand throughout this section for the relation of instantiation provided by the model.

\[
(69) \quad \text{For all } j, k \in E: E \mid j \leq_{J(E)} E \mid k \iff j \leq_E k
\]

It is easy to show that \( J(E) \) and \( \leq_{J(E)} \) jointly constitute a mereology. Indeed, this mereology is demonstrably isomorphic to the one constituted by \( E \), the universe of discourse, and \( \leq_E \), the relation of instantiation. As might be expected, overlap and sums in \( J(E) \) are defined in terms of overlap and sums in \( E \):

\[
(70) \quad \text{For all } j, k \in E: E \mid j \circ_{J(E)} E \mid k \iff j \circ_E k.
\]

\[
(71) \quad \text{For all } j \in E \text{ and for all nonempty } F \subseteq E: E \mid j \text{ Su } \{ E \mid k: k \in F \} \iff j \text{ Su } F
\]

Are there any expressions of natural languages which select their denotations from among the elements of \( J(E) \) and the operations of its algebra? To answer this question let us return to the case in which all the ink of the world is found in two distinct containers; all the ink in one container is red, while all the ink in the other is black. Consider now the sentence in (72).

\[
(72) \quad \text{The ink is either red or black.}
\]

There is a sense in which (72) is false. Notice, for instance that it cannot be paraphrased as indicated in (73). Quite simply, the mereological sum of the world’s ink is neither red nor black.

\[
(73) \quad \text{The ink is red or the ink is black.}
\]

But there seems to be another sense to (72)—one which in effect asserts that the total portion of the world’s ink is the mereological sum of the red ink and the black ink. Taken in this sense, (72) is obviously true.

To allow for this second sense, we will let some adjectives of natural language denote domains in the universe of discourse. Let us assume then that some adjectives may select their denotations from among the elements of \( J(E) \). Now \textit{red} and \textit{black} may each denote an element of \( J(E) \). But let us also assume that \textit{not}, \textit{or}, and \textit{and} may denote the operations of the mereology constituted by \( J(E) \) and \( \leq_{J(E)} \), so that the complex predicate \textit{red or black} can be interpreted as indicated in (74). Here \( r \) stands for the red and \( b \) for the black.

\[
(74) \quad \llbracket \text{red or black} \rrbracket = \llbracket \text{red} \rrbracket +_{J(E)} \llbracket \text{black} \rrbracket = E:\text{red} +_{J(E)} E:\text{black}
\]
The sentence in (72) is now predicted to be ambiguous. It may assert either (75a) or (75b).

(75) a. \[[\text{the ink}]] \in [\text{red}] +_{P(E)} [\text{black}]

b. \[[\text{the ink}]] \in [\text{red}] +_{J(E)} [\text{black}]

(75a) states that the kind 'ink' is either in the set of instances of the kind 'red' or else in the set of instances of the kind 'black'. This statement is false in the state of affairs in question. (75b), on the other hand, states that the kind 'ink' is in the set of instances of the kind constituted by the kind 'red' and the kind 'black' (see (74) above). As desired, this statement is true in the state of affairs in question.

It should not escape the reader that the ambiguity stems not from an ambiguity in the denotation of the simple predicates (\text{red} and \text{black} could denote domains in both cases). Rather, the ambiguity arises from the way these simple denotations combine into a complex denotation (either through a sum taken in \(P(E)\) or through a sum taken in \(J(E)\)).

The interpretation in (75b) has been all but taken from Roeper (1983). But similar interpretations have been advanced in Lønning (1987). Consider a universe in which there is coffee. Let us say now that some (but not all) of the coffee is hot, and that some (but not all) of the coffee is black. Let us suppose now that the situation changed: the black coffee disappeared (but the rest of the coffee did not). Consider now (76).

(76) The hot coffee did not disappear.

As it turns out, (76) exhibits a scope ambiguity. If negation has wide scope over the definite article then the assertion is (77a). If negation has narrow scope, the assertion is (77b) instead.

(77) a. \[[\text{the hot coffee}]] \in [\text{disappear}]'_{P(E)}

b. \[[\text{the hot coffee}]] \in [\text{disappear}]'_{J(E)}

The assertion in (77a) is true under the given circumstances (the kind 'hot coffee' is an element of the set of portions which did not disappear). The assertion in (77b), on the other hand, is false under the given circumstances (the kind 'hot coffee' is not an instance of the kind of portions which did not disappear).

It should again be noted that the ambiguity in (77) does not stem from an ambiguity in the denotation of the simple predicate \text{disappear} (this predicate may denote the domain of the black coffee in both cases). Rather, the ambiguity arises from the way this simple denotation becomes complex (either by taking its complement in \(P(E)\) or by taking its complement in \(J(E)\)):
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(78) a. [(disappear)]_p(E) = (E \mid b)^p(E) = E \mid c - p(E) E \mid b
b. [(disappear)]_j(E) = (E \mid b)^j(E) = E \mid c - j(E) E \mid b = E \mid b'

Note that b stands here for the black coffee, b' stands for the nonblack coffee, and c stands for the coffee.

Now, as Lønning (1987) would have it, the ambiguity of (76) argues that every mass noun should denote a portion of stuff (rather than a set of portions of stuff). Such, however, is not necessary. To account for the ambiguity it will be sufficient to assume that the set of individuals provided by a model constitute a mereology with respect to the relation of instantiation provided by the model. This will ensure the existence of a second mereology (the mereology of domains in the universe). The operations of this second mereology can then be used to interpret complex predicates in order to generate a second reading for (76) as desired. It is interesting to note that to account for the ambiguity in (76), Lønning proposes a branching in the logical type of nouns, while we propose a branching in the interpretation of operators.

22 Uncountability and Plurality

Plurals and uncountables have been often noted to exhibit intriguing similarities. "Plural count nouns pattern like singular mass nouns in all significant respects," writes McCawley (1968, 568), "they take a zero indefinite article, they both participate in a partitive construction, and in English they both take a zero generic article rather than the generic the of singular nouns." For our part, we have seen that plurals and uncountables are similar in that they have cumulative reference, may be quasuniuniversally determined by most, tolerate collective predication, fail to support a presupposition of uniqueness with the definite article, and contain the kinds they constitute. In each one of these respects they contrast with singulars, which fail to have cumulative reference, cannot be determined by most, and so on.

Notice that the similarities between plurals and uncountables are far from being idiosyncratic to English. In Sinhalese, plurals and uncountables share morphology (cf. Allan 1980, 542). And similar situations hold outside the Indoeuropean family of languages. In the Austric language Manam, all mass nouns are said to be plural (cf. Barlow 1988, 51); in Lingala and other Bantu languages, noun classes can be found which are entirely constituted by plurals and uncountables—which will therefore bear the same distinctive prefix and determine the same concordance on their adjuncts (cf. Mufwene 1980).

It seems clear that the similarities between plurals and uncountables call for a principled account. But one such account has
been forthcoming under our proposals. For, given that both plurals and uncountables denote subdomains in the universe, nonvacuous plurals and uncountables will *ipso facto* tolerate *most*. In addition, they will denote complete sets. Consequently, their reference will be cumulative. In particular, they will contain the kinds they constitute. They will therefore support a presupposition of uniqueness if they contain nothing but the kinds they constitute; they will otherwise tolerate collective predication.

Singulars, on the other hand, denote pairwise disjoint sets. Consequently, they can only denote *degenerate* subdomains (subdomains with at most one element). As to the differences between plurals and uncountables, they have been seen to be derived from the claim that they denote subdomains with different mereological structures. As has been illustrated throughout, the denotations of plurals are atomistic, whereas the denotations of uncountables are atomless.

To account for the similarities between plurals and uncountables, Mufwene (1980) endowed linguistic theory with an opposition between the singular (or INDIVIDUATED) and the plural/uncountable (or NONINDIVIDUATED). As we see it, individuation is pairwise disjointness. When seen in this light, the individuated and the nonindividuated are opposed in the main, but overlap in the degenerate cases created by plurals and uncountables which denote sets with at most one element.
On the Semantic Character of Nouns

1 Introduction

We have said that a kind is atomistic if it is entirely constituted by atoms of the universe. But we could have said, without loss of generality, that a kind is atomistic if every instance thereof has an atom as instance. More formally,

(1) **Atomistic Kinds**

Let $E$ be a universe and let $\leq$ be its relation of instantiation. Any $k \in E$ is atomistic if and only if every $j \in E$ is such that $j \leq k$ implies that there is an atom $a$ of $E$ such that $a \leq j$.

We have also said that a kind is atomless if it is not instantiated by any atom of the universe. Alternatively, we could have said that a kind is atomless if no instance thereof has an atom as instance. In more formal terms,

(2) **Atomless Kinds**

Let $E$ be a universe and let $\leq$ be its relation of instantiation. Any $k \in E$ is atomless if and only if no $j \in E$ is such that $j \leq k$ implies that there is an atom $a$ of $E$ such that $a \leq j$.

Atomistic and atomless kinds are different in one fundamental respect: atomistic kinds can be individuated ('atomized') in the universe, whereas atomless kinds cannot. Yet, atomistic and atomless kinds are also similar in one important respect. Both are mereologically homogeneous (or *homomeric*) in the sense that every instance of an atomistic kind is itself atomistic, and every instance of an atomless kind is itself atomless. We may therefore define

(3) **Homomorphic Kinds**

Let $E$ be a universe. Any $k \in E$ is homomorphic if and only if it is either atomistic or atomless.

It should be clear that no kind can be both atomistic and atomless. This means that atomisticity and atomlessness, as applied to kinds, are logical contraries. The application of these notions to kinds thus differs to their application to mereologies. For, as we have seen above, there is
one mereology which is both atomistic and atomless: the empty mereology.

But atomisticity and atomlessness, as applied to kinds, are only logical contraries; they are not logical contradictories. This means that even though there can be no kind which is both atomistic and atomless, there may well be some kind which is neither atomistic nor atomless. To illustrate, let us consider a universe which contains both an atomistic kind and an atomless kind. Since universes are complete, the sum of these two kinds must also belong to the universe. But notice now that this sum will not be atomistic, as no instance of the atomless kind will have an atom as instance. But the sum will not be atomless either, as every instance of the atomistic kind will have an atom as instance.

Let us consider now any sum of an atomistic kind and an atomless kind. There is a clear sense in which such sums are mereologically heterogeneous (or heteromeric). We will thus define

(4) \textit{Heteromeric Kinds}

Let $E$ be a universe. Any $k \in E$ is heteromeric if and only if it is neither atomistic nor atomless.

It should be clear that no kind can be both homomeric and heteromeric. Homomericity and heteromericity are thus logical contraries. But every kind is either homomeric or heteromeric. It follows that homomericity and heteromericity are not only logical contraries, but also logical contradictories. The set of kinds contained in an arbitrary universe can therefore be partitioned as follows.

(5)

\begin{center}
\begin{tikzpicture}
    \node (kinds) {kinds}
    \childrarline {homomeric} {atomistic}
    \childrarline {heteromeric} {atomless}
    \childrarline {atomic} {polyatomic}
\end{tikzpicture}
\end{center}

Naturally, (5) is also a partition of the arbitrary universe itself, since every individual is a kind and every kind is an individual (as will be recalled, the relation of instantiation is reflexive).

But it might be thought that no universe can contain both atomistic and atomless kinds. If this were so, then every kind would be homomeric and no kind would be heteromeric. Homomericity and heteromericity
would thus become trivial properties of kinds. It can be shown, however, that universes may indeed tolerate mereological diversity. It can be shown, in fact, that given any atomistic universe and given any atomless universe, a universe can be constructed which contains both as subsets.

2 Nominality as Homomericity

But the tolerance of mereological diversity in a universe need not be the same as tolerance of mereological diversity in a noun. The purpose of this section is to claim that the latter is much more limited than the former. Specifically we wish to propose

\[ (6) \quad \text{The Homeric Condition} \]

\[ \text{Every nominal stem denotes a set of a homeric individuals.} \]

Notice that (6) places a strong constraint on the semantics of nouns. Consider first singular countable nouns. As constrained in Chapter 4, these nouns can only denote pairwise disjoint subsets of the universe. When (6) is brought to bear, singular countable nouns can only denote pairwise disjoint subsets of the homemic portion of the universe.

Or consider plural nouns. As constrained in Chapter 4, these nouns can only denote atomistic subdomains in the universe. When this constraint is coupled with (6), plural nouns can only denote the atomistic subdomains of the homemic portion of the universe. Cast in terms of kinds, a singular noun may be said to denote the partition of a homeric kind; a plural noun will denote one of the atomistic subdomains of a homeric kind (interestingly, every uncountable noun must denote a subdomain in the homeric portion of the universe simply by virtue of denoting an atomless subdomain; an uncountable noun will always denote an atomless subdomain of a homeric kind; the Homemic Condition does not constrain the denotation of uncountables beyond what was said in Chapter 5).

Incidentally, recall that a set of atomistic individuals may only constitute an atomistic subdomain, whereas a set of atomless individuals may constitute either an atomistic or an atomless subdomain. To see this, the reader should recall the two subdomains of the kind ‘wine’ discussed in the preceding chapter. The first was the atomless subdomain generated by ‘wedge shaped’ portions of wine; the second was the atomistic subdomain generated by the three main varieties of wine (the red, the white, the rosé).

It should be emphasized that the Homemic Condition is far from being a trivial constraint on the predicates of natural languages. Consider for instance the adjective red. This adjective can clearly be
true both of atomistic individuals like schoolhouses and of atomless individuals like portions of ink. It can moreover be true of kinds having schoolhouses and portions of ink as instances. But such kinds are heteromeric. The adjective \textit{red} can therefore be true of heteromeric kinds, and the condition in (6) cannot be generalized to adjectives.

What has been said of the adjective \textit{red} will obviously hold of the verb phrase \textit{is red}, of the relative clause \textit{which is red}, and of the prepositional phrase \textit{of red color}. It follows that the condition in (6) cannot be extended to verb phrases, relative clauses, or prepositional phrases. The Homomeric Condition therefore presents itself as a substantive characterization of nouns on purely semantic grounds.

But this is not to say that the Homomeric Condition constitutes a complete characterization of the set of nouns. Consider an adjective like \textit{potable}. It seems natural to interpret this adjective as a set of atomless (and hence homomeric) individuals, namely the set of liquid portions of the universe which are suitable for human consumption. Or consider the adjective \textit{tall}. Such an adjective should be taken to denote a set of atomistic (hence once again homomeric) individuals, namely the set of atomistic kinds every atomic instance of which is tall. Although \textit{necessary}, the Homomeric Condition is not \textit{sufficient} for nounhood.

3 Some Apparent Exceptions

But consider now \textit{redness}. The foregoing does not establish this noun as \textit{aprimafacie} counterexample to the Homomeric Condition, since \textit{red} and \textit{redness} are simply not synonymous (schoolhouses and ink may be red even though they are certainly not redness). In fact, as we see it, \textit{redness} denotes a particular set of homomeric individuals, namely the set of instances of being red (this is the set which includes the properties of being red now, being red here, being red while seeming to be orange, and so on).

To be more precise, let us assume that properties may be elements of the universe.\footnote{See Chierchia (1985) and work subsequent thereto.} Let us moreover assume that the relation of instantiation over $\llbracket \textit{redness} \rrbracket$ is part of the relation of instantiation provided by the model. Our view is that \textit{redness} denotes an atomless domain in the universe. It denotes, therefore, a set of homomeric individuals—thus abiding by (6). Notice that the atomlessness of $\llbracket \textit{redness} \rrbracket$ accounts for its behavior as an uncountable. Thus, \textit{redness} cannot be properly pluralized, numbered, described in terms of size, described in terms of shape, or quantified in individual terms. We illustrate with (7).
(7) a. There isn’t much redness left in the world.
   b. ?There aren’t many rednesses left in the world.

But consider now *succotash*, the noun which denotes the philosophically interesting mixture of corn and lima beans brought to the fore in Sharvy (1979). Let us suppose for the sake of argument that *succotash* denotes as indicated in (8).

(8) $$[[\text{succotash}]] = \{x+y: x \in [[\text{corn}]] \land y \in [[\text{lima beans}]]\}$$

If $$[[\text{corn}]]$$ is a set of atomless individuals, and if $$[[\text{lima beans}]]$$ is a set of atomistic individuals, then $$[[\text{succotash}]]$$ would be a set of sums of atomistic and atomless individuals. It would therefore be a set which is entirely constituted by heteromeric individuals—thus flying in the face of (6).

But *succotash* simply cannot denote as indicated in (8). Notice first that this noun behaves like an uncountable in every respect: it cannot be properly pluralized, numbered, described in terms of size, described in terms of shape, or quantified in individual terms:

(9) a. There wasn’t much succotash left in his plate.
   b. ?There weren’t many succotashes left in the plate.

But to behave like an uncountable, *succotash* must denote an atomless subdomain in the universe—something which $$\{x+y: x \in [[\text{corn}]] \land y \in [[\text{lima beans}]]\}$$ is not. Rather, *succotash* must denote as indicated in (10), where $$s$$ stands for the atomless kind which is constituted by all the succotash in the universe of discourse.

(10) $$[[\text{succotash}]] = E \upharpoonright s = \{x \in E: x \leq s\}$$

The denotation of *succotash* thus contrasts with that of *corn and lima beans*, which should be as follows.

(11) $$[[\text{corn and lima beans}]] = \{x+y: x \in [[\text{corn}]] \land y \in [[\text{lima beans}]]\}$$

As Sharvy (1983, 234) put it,

A predicate like ‘is beer and wine’ is interpreted as a structure which turns out to be isomorphic to the algebraic direct product of the [structures] corresponding to ‘is sm beer’ and ‘is sm wine’. The ‘and’ in ‘is (sm) beer and wine’ cannot be logical conjunction—nothing is both beer and wine. Rather, it expresses summation: the predicate is satisfied by any sum of sm beer plus sm wine (not necessarily mixed).²

² Notice that $$[[\text{is (sm) beer and wine}]]$$, call it $$A$$ for short, is not closed under complementation relative to $$\Sigma(A)$$. For consider any element of $$A$$ which contains the kind ‘beer’ as instance. No such element can be complemented in $$A$$, as no element of $$A$$ will be disjoint with it. But this means that $$A$$ cannot be a
Needless to say, the analysis in (11) presents no problem for the Homomeric Condition, as the expression *corn and lima beans* is hardly a noun.

But further evidence against the interpretation in (8) can be provided. For, notice that (8) implies that every quantity of succotash must contain both corn and lima beans. Yet, one can truthfully say of Johnny that he did not finish the succotash in his plate if he ate all the corn but left three lima beans.

We conclude that *succotash* does not counterexemplify the Homomeric Constraint; it does not denote the sum of heteromeric quantities constituted by corn and lima beans. Yet, one would still like to get an idea of how the meaning of *succotash* is related to the meaning of *corn* and the meaning of *lima beans*. This can be done by a meaning postulate—one which asserts, for instance, that succotash is produced by mixing corn with lima beans (which does not imply that succotash will remain the said mixture).3

### 4 Nominality as Atomlessness: Classifier Languages

Although the constraint in (6) is strong in itself, there are quite a few languages which place even stronger constraints on the mereological diversity tolerated by their nouns. It is to them that we shall now turn.

Let us begin by defining a *classifier language* as one in which the combination of a cardinal adjective and a noun must be mediated by an expression which “suppl[ies], or presuppose[s], a principle for individuating [the] entities [denoted by the nouns] and grouping them into kinds.”4 Consider for instance the Tzeltal phrases in (12).

(12) a. ʔoʃ tehk te?
   three plant tree
   ‘three trees’ (*lit.* ‘three plants of tree’)

b. čan tul winik
   four human man
   ‘four men’ (*lit.* ‘four humans of man’)

mereology—and this even if [[beer]] and [[wine]] are. A can only be a quasi-mereology (see Chapter 2, Note 3). Similar points can be made with [[corn and lima beans]] and with [[are men and women]]. See Sharvy (1980, 621f).

3 Similar solutions will be called for, it seems, to relate the meaning of *brass* to the meanings of *copper* and *tin*.

4 Being a classifier language is actually a matter of degree. Thus, Allan (1977, 286) notes that some Burmese and Vietnamese nouns call for the criterial mediating expression whereas some do not. Individuating expressions may also be omitted, at least in part, in colloquial Khmer and Thai. See Lyons (1977, 460ff), on which we base our discussion.
Here *tehk* and *tul* 'supply or presuppose a principle of individuation'. Since these expressions cannot be omitted from (12) or related constructions, Tzeltal is indeed a classifier language. Tzeltal thus differs from English, which does not call for any explicit mention of units in this context. The set of classifier languages has members which differ from each other as much as Tzeltal, Bengali, Mandarin, and Kiriwina (cf. Allan 1977).

Classifier languages are so called because the expressions which supply or presuppose their principles of individuation are called *classifiers*. Classifiers in turn owe their name to the fact that different classifiers must be used with different nouns. Thus Tzeltal uses *tehk* for plants, *tul* for humans, *koht* for animals, and so on (cf. Berlin 1968). But the name given to classifiers should not obscure the fact that their main function is to provide a principle of individuation; that different classifiers are used with different nouns is but a consequence of the fact that different domains usually call for different principles of individuation.5

Let us turn now to the Tzeltal phrase in (12a). We will suppose that the classifier *tehk* 'plant' is an expression which denotes, not the set of plants in the universe, but rather the set of plant contents in the universe. We will moreover suppose that the noun (*te* 'tree' is uncountable, and denotes the set of tree portions in the universe. Equipped with these two assumptions, the denotation of our phrase may proceed straightforwardly as follows.

\[
(13) \quad \llbracket \text{tehk } \text{te} \rrbracket = \llbracket \text{te} \rrbracket \cap \llbracket \text{te} \rrbracket
\]

To illustrate, let us imagine that all the tree content in the universe is to be found in four plants. Call these plants \(a, b, c, d\). We may now say that the tree content in these plants is \(m(a), m(b), m(c), m(d)\). Notice that these tree contents must be pairwise disjoint. They will therefore be the atoms of an atomistic subdomain. Such is the subdomain diagramed in (14).

Let us say now (solely for the sake of illustration) that every plant in the universe is a tree. Relative to this model, our phrase will denote as indicated in (15), where \(E \subseteq m(a+b+c+d)\) is the set of tree portions. The denotation of (12a) will thus be the set enclosed in (16).

---

5This is not to say that there is a free association of nouns and their classifiers. "To some extent," writes Lyons (1977, 466), "the principles of sortal classification, in so far as they are discernible, appear to be universal, being based upon the ontological salience of natural kinds and the perceptual or functional salience of certain criterial attributes."
But if this is so, the idiomatic gloss ‘three trees’ of our phrase is misleading in one important respect. Contrary to what the English gloss may suggest, the Tzeltal phrase does not denote a set of atomistic \textit{kinds}; it denotes only a set of atomless kinds which are part of an atomistic \textit{structure}. This structure, which is the one in (14), will not be identical to \{a, b, c, d\}*. It will only be mereologically isomorphic to it.\footnote{Recall Chapter 5, Section 4, where we noted the isomorphism which holds between the mereology generated by the set \{a, b, c\} of gold rings and the one generated by the set \{m(a), m(b), m(c)\} of gold in those rings.}

There is a respect, then, in which ‘three plants of tree’, the literal translation of (12a) is to be preferred to ‘three trees’, the more idiomatic gloss of the phrase. As remarked in Allan (1977, 293), “it might be both more realistic and revealing to translate [the Thai example in (17)] as ‘three members of the teaching profession.”
We thus take issue with the characterization of classifiers set forth in Krifka (1987, 9f), where classifier constructions like the ones in (12a) and (17) are taken to be synonymous with their idiomatic English glosses.

Crucial to the foregoing has been the assumption that the classifier tehk denotes not the set of plants in the universe, but rather the set of plant contents in the universe. For suppose to the contrary that the classifier were to denote simply the set of plants in the universe. As (13) would have it, we would have to take the intersection of a set of atomistic kinds (the one denoted by the classifier) and a set of atomless kinds (the one denoted by the classified noun). Since atomisticity and atomlessness, as applied to kinds, are logical contraries, this intersection would necessarily amount to the empty set, and the phrase in (12a) would necessarily denote the empty set. This would in turn lead us to predict, incorrectly, that the phrase is contradictory (hence ill formed).

Naturally, this is not an idiosyncracy of tehk. If the straightforwardness of (13) is to be emulated by other interpretations, analogous denotations for every classifier are in order. One way to ensure this would be to stipulate that every classifier must denote a set of atomless kinds. But
classified nouns must also denote sets of atomless kinds.\(^7\) A generalization is surely being missed. As we see it, the generalization is that classifier languages abide by (18), a more stringent version of the Homomeric Condition.

(18) \textit{The Atomless Condition}

Every nominal stem denotes a set of atomless individuals.

All we need to stipulate now is that classifiers are countable nouns.\(^8\) Classifiers will thus denote atomistic subdomains whose elements are all atomless. Now classifiers may overlap with the nouns they properly classify. In fact, this overlap will itself be an atomistic subdomain in the universe. The denotation of a cardinal adjective can then apply to such a subdomain. As seen in (13), this was how the denotation of a classifier construction was indeed produced.

But Greenberg has observed that the solidarity which holds between a numeral and a classifier is greater than the one which holds between either of these and the classified noun:

We have seen what might be called, anthropomorphically, the aversion of collectives to direct construction with a numeral and the intervention of an individuated noun, the classifier, as one of the devices to avoid this direct confrontation. This aversion has, therefore, as its natural counterpart, the corresponding attraction to the classifier and an immediate constituent structure in which the numeral goes directly with the classifier while the numeral+classifier combination as a whole enters into a more remote construction with the enumerated noun (Greenberg 1977, 293; emphasis supplied).

Evidence for the constituent structure suggested by Greenberg comes from the fact that, of the six possible word orders among the three

\(^7\) As pointed out in Allan (1977, 294), the noun of a classifier construction 'has the characteristic of a mass, collective, or uncountable noun'. True, Allan also mentions that Yucatec Mayan and Algonquian allow plural nouns in their classifier constructions. But the plurals of the former, we hasten to add, are optional. More importantly, they have no discernible semantic effect. They may therefore be regarded as the semantically null plural of uncountables like oats and clothes. As to the plurals of Algonquian, they must be present 'where the classifier counts discrete objects, but [...] absent where the translation into English is a partitive expression'. This means that the nouns of the true (i.e. semantic) classifier constructions of Algonquian may still be regarded as uncountable; in any event, they are never plural.

\(^8\) And that their inflections, if there are any, are semantically void (just as we argued were the inflections called for by the cardinal adjectives of English). It is interesting to note in this regard that Greenberg (1977, 286) has observed (after unpublished work by Mary Sanches) that '[n]umeral classifier languages generally do not have compulsory expression of nominal plurality, but at most facultative expression'.
elements Q(quantifier), Cl(classifier), and N(oun), only the four in (19) actually occur. As Greenberg (1977, 293) points out, "[t]he two non-occurring orders Cl-N-Q and Q-N-Cl have the property that the quantifier and the classifier are separated by the head noun." 9

(19) a. Q-Cl-N: Bengali, Chinese, Vietnamese, Amerindian and Semitic languages.
 c. Cl-Q-N: Kiriwina.
 d. N-Cl-Q: Louisiade Archipelago

In fact, the attraction of the numeral to the classifier is such that they may even share an accent, thus leading many analysts to regard the numeral+classifier construction as a single word.

In light of this, the analysis of (12a) provided in (15) may be called to question on the grounds of compositionality. As (15) would have it, the two noun denotations combine before the adjectival denotation is applied. The syntactic evidence suggests, on the other hand, that the adjective must combine with the classifier noun before the construction combines with the classified noun. Fortunately, the same semantic result can be arrived at by a different syntactic route. Compare (15) above and (15') below.

\[
(15') [\text{?o§ tehk te}^9] = [\text{?o§}]([\text{tehk}]) \cap [\text{te}^9]
\]
\[
= [\text{?o§}] (\{m(a), m(b), m(c), m(d)\})^* \cap E \mid m(a+b+c+d)
\]
\[
= \{m(a+b+c), m(a+b+d), m(a+c+d),
\]
\[
m(b+c+d) \cap E \mid m(a+b+c+d)
\]
\[
= \{m(a+b+c), m(a+b+d), m(a+c+d),
\]
\[
m(b+c+d)\}
\]

Here indeed the cardinal adjective applies to the classifier and selects the set of three-plant contents. This set is then intersected with the set of tree portions of the universe. Since we have assumed that every plant in this universe is a tree, the set of three-plant contents survives the intersection unscathed.

It should be noted that semantically, ?o§ is indistinguishable from the trial number inflection as interpreted under (10) in Chapter 4. Numerals can therefore be analyzed as number inflections in languages whose numeral+classifier construction is regarded as a single word—provided, that is, that the 'generalized' interpretation of number inflection we have proposed is made truly general.

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9 The assignment of languages to word order types has been taken from Allan (1977, 288).
It should also be noted that modes of combination employed in (15) and (15') do not have to be stipulated *ad hoc*. Given the Intersection Principle and the Functional Principle of earlier chapters, such are the only possibilities; the interpretations in (15) and (15') represent the only ways to combine the meaningful parts involved therein into a meaningful whole. It is therefore pleasing that they should yield the same result.
On Conceptional Neuterality

1 Introduction

In one of the most intriguing sections of his *Philosophy of Grammar*, Otto Jespersen (1924, 241ff) proposed a linguistic category he called the conceptional neuter. According to Jespersen, the conceptional neuter can be found in many languages, but is not constituted, in any one of them, by more than a handful of mostly pronominal expressions. Thus, instances of what Leo Spitzer called *das grosse Neutrum der Natur* (e.g. the pronouns in English *it rains*, German *es regnet*, and French *il pleut*) are conceptionally neuter. Also conceptionally neuter are the idiomatic *it of take it easy*, the impersonal pronoun of French *on frappe à la porte*, the quantifying pronouns *what, nothing, everything, something*, and the "adjectives in the generic" as in English *the beautiful*, or Spanish *lo bello* 'what is beautiful'. Finally, the anaphors of predicates and propositions are also conceptional neuters. To illustrate the former, Jespersen points to the Spanish *lo* 'so' in *personas que parecen buenas y no lo son* 'people who seem good and are not so'. To illustrate the latter he invokes the English *that* in *Can you forgive me? Yes, that is easy enough*.

It is clear that as far as Jespersen is concerned, these expressions constitute a natural semantic class. But if indeed they are, what is their distinctive trait? Herein lies the intrigue. Notice that the term 'conceptional' will be of little help. Jespersen readily acknowledges that this term was chosen only "for want of a better term." And the terms with which 'conceptional' is used interchangeably (viz. 'real', 'notional', 'natural', 'universal', 'unspecified') are rather disconcerting, both in their variety and in their imprecision.

But the term 'neuter' is not very helpful either, as *The Philosophy of Grammar* applies this term to two other categories. One is "the specified or concrete neuter which we have when in English we refer to a previously mentioned house or worm, etc., as *it*." The other one is "the arbitrary neuter which we have when in German we refer to a previously mentioned *haus* or *mädchen* as *es* because the word happens to be of
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the neuter gender." It is never made clear, however, what these categories share with the conceptional neuter.¹

The conceptional neuter was briefly mentioned in Brøndal (1928, 117), where the category of neuter pronouns is described as the most abstract of pronominal categories. In the mind of this grammarian, the abstractness of this category accounts both for its extreme underrepresentation in the languages of the world and for its considerable syntactic variation. As will be seen below, there is a clear sense in which the class of conceptional neuters is \textit{la plus abstraite}. And conceptional neuters may indeed be found in a wide variety of grammatical paradigms. Yet, we believe that they are represented in all the languages of the world, even though they are only infrequently \textit{marked contrastively} as such.

More recently, Priestly (1983, 349) has characterized conceptional neuterality as 'impersonality' and as one of the most resilient uses of a neuter. In addition, this grammarian finds further instances of the conceptional neuter in the Lithuanian neuter adjective (cf. \textit{gēra} 'good' in \textit{gēra eina toli} 'good goes far'), in the Romance reflexes of Latin \textit{ecce hoc} 'this' (cf. French \textit{ce}, Italian \textit{ciò}, Catalan \textit{això}), in the Scots Gaelic pronoun 's, which translates either as 'it' or as 'that', but in any case "refers to statement only, and is not a pronominal replacement for noun phrases' (cf. Scots Gaelic 's \textit{eadh} 'that is so'), and in the Khotanese expression \textit{dtumā} 'I entered', which literally means 'it was entered by me'.

Unfortunately, Priestly provides us with no elucidation of his term 'impersonality' (the etymological analysis of 'impersonal' as 'nonhuman' does not seem to be the intended sense of this ubiquitous term). And the same problem afflicts the proposals in Klajn (1985), who characterized the various kinds of conceptional neuters as partially different manifestations of 'indefinite reference' arising as a consequence of the unmarked (neither male nor female) nature of the neuter. For, notice that the present use of 'indefinite reference' cannot be the traditional one. Personal and demonstrative pronouns have been

¹ The conceptional neuter can be part of a grammatical neuter—as it is in Latin. It can moreover be indistinguishable from such a gender—as it is in Romansch. Yet, it is entirely possible for our presumed category not to be a grammatical neuter. In fact, the conceptional neuter can even be a grammatical gender which is defined by its opposition to a grammatical neuter—as it is in a number of Slavic languages, possibly including Russian. See Hale & Buck (1903, §325). See also Corbett (forthcoming) and the references cited therein.
traditionally taken to be prototypically definite. Yet, they are among the prime examples of the conceptional neuter.

In our view, the key to conceptional neuterality is to be found in *A Modern English Grammar on Historical Principles*, where Jespersen argued that some of the pronouns he would later consider conceptionally neuter, were in fact uncountables:

*Something great* refers to a 'mass'; and has no plural, *some great thing* has the plural *some great things*, referring to 'countables' [...] the difference between *nothing new* and *no new thing* corresponds to the distinction between mass-words (non-countables) and thing-words (countables) [...] This explains the distinction [made in] *I have not done anything good nor said any good thing* [...] Like other mass words, *nothing* may be combined with *much* [though not with *many*] (cf. Jespersen 1954, II, §§5.213, 17.323f).

Although conceptional neuters are not quite mass terms, they are altogether like uncountables in that they provide no criterion for the individuation of their reference. The purpose of this chapter is to build on the mereological taxonomy of kinds presented in the preceding chapter and to argue that the conceptional neuter is a natural semantic class whose distinctive trait lies with the ability to involve the entire universe of discourse. Conceptional neuters thus contrast with their nonneuter counterparts, which will be seen to denote within mereologically restricted portions of the universe of discourse.

To make this somewhat more concrete, let us consider (1), an abbreviated version of the mereological taxonomy of kinds which was presented in the preceding chapter. The abbreviatory conventions we have adopted here are the following. \(A\) is the set of atomic kinds. \(M\) is the set of atomistic (or molecular) kinds. \(H\) is the set of homomeric kinds. \(E\) is the set of all kinds (which is the universe of discourse). \(M - A\) is the set of polyatomic kinds (= the difference between the set of atomistic kinds and the set of atomic kinds). \(H - M\) is the set of atomless kinds (= the difference between the set of homomeric kinds and the set of atomistic kinds). \(E - H\) is the set of heteromeric kinds (= the difference between the set of kinds and the set of homomeric kinds).

It will be argued that neuters may draw their denotations from within \(E\), the entire universe of discourse, whereas nonneuters must select theirs from within \(H\), the homomeric portion of the universe of discourse. In fact, the denotational reach of some nonneuters is even confined to \(A\), the atomic portion of the universe of discourse.

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2 The reader should beware that \(M\) is intended here as mnemonic for 'molecular' rather than 'mass'. The set of atomless individuals is \(H - M\) rather than \(M\).
Our approach will be to concentrate on Spanish, a language whose opposition between (conceptional) neuterality and nonneuterality is expressed overtly in a wide range of pronominal paradigms. The overt expression of this contrast will enable us to identify the instances in which conceptional neuters are possible while their minimally different counterparts are not. The pervasive expression of this contrast will allow us to corroborate such identifications. It will then be expected—though at no point assumed—that our characterization of the conceptional neuter of Spanish will apply with full crosslinguistic generality, so that the conceptional neuter may be regarded as a category of universal grammar constituted by pronouns which can denote without regard to mereology.

2 The Spanish Forms

It has become customary among grammarians to recognize three genders in Spanish and to refer to them as the masculine, the feminine, and the neuter. These three genders have been taken to be distinctively marked in pronouns which constitute the stressed and nonstressed paradigms of personal pronouns; in the proximal, medial, and distal paradigms of demonstrative pronouns, and in the existential and nonexistential paradigms of indefinite pronouns. A tabular representation of these descriptive decisions is provided in (2).

Although nowadays limited to the paradigms in (2), the overt distinction between masculine, feminine, and neuter used to obtain in further pronominal paradigms. Consider for instance the paradigm constituted by the masculine *aquéste*, the feminine *aquésta*, and the

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3 We follow Bello (1860, §§277, 302, 324, 969, Nota V), Hanssen (1913, §545), Fernández Ramírez (1951, §§71f, 141, 158), Hall (1965), Pottier (1972, 146), Lázaro Carreter (1975, §13), Luján (1980, §4.2), Ojeda (1982, 1983), and others in regarding *el, la, lo* as pronouns. We follow Hottenroth (1982) and others in regarding the three demonstratives of each gender as proximal, medial, and distal. We follow universal practice in regarding the indefinite paradigm as containing existential and nonexistential forms.
neuter *aquesto*. These were the 'emphatic' versions of the proximal pronouns in (2). Illustrative uses of these pronouns are provided in (7).

(2)

<table>
<thead>
<tr>
<th></th>
<th>MASCULINE</th>
<th>FEMININE</th>
<th>NEUTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRESSED</td>
<td><em>él</em></td>
<td><em>ella</em></td>
<td><em>ello</em></td>
</tr>
<tr>
<td>NONSTRESSED</td>
<td><em>el</em></td>
<td><em>la</em></td>
<td><em>lo</em></td>
</tr>
<tr>
<td>PROXIMAL</td>
<td><em>éste</em></td>
<td><em>ésta</em></td>
<td><em>esto</em></td>
</tr>
<tr>
<td>MEDIAL</td>
<td><em>ése</em></td>
<td><em>ésa</em></td>
<td><em>eso</em></td>
</tr>
<tr>
<td>DISTAL</td>
<td><em>aquél</em></td>
<td><em>aquélla</em></td>
<td><em>aquello</em></td>
</tr>
<tr>
<td>EXISTENTIAL</td>
<td><em>alguno</em></td>
<td><em>alguna</em></td>
<td><em>algo</em></td>
</tr>
<tr>
<td>NONEXISTENTIAL</td>
<td><em>ninguno</em></td>
<td><em>ninguna</em></td>
<td><em>nada</em></td>
</tr>
</tbody>
</table>

Illustrative uses of the contrasts displayed in this table can be found in (3)—(6) below. It should not escape the reader that the contrast between the neuter and the nonneuter is not only one of form, but also one of meaning.

(3)  
  a. *Él* es bueno.  
      'He is good.'
  b. *Ella* es buena.  
      'She is good.'
      'It is good.'

(4)  
  a. *El* que es bueno destaca.  
      'The one (masc.) who is good stands out.'
  b. *La* que es buena destaca.  
      'The one (fern.) who is good stands out.'
  c. *Lo* que es bueno destaca.  
      'What is good stands out.'

(5)  
  a. *Ése* está pesado  
      'That thing/stuff (masc.) is heavy.'
  b. *Ésa* está pesada.  
      'That thing/stuff(fem.) is heavy.'
  c. *Eso* está pesado.  
      'That is heavy.'

(6)  
  a. *Alguno* bueno ha de haber.  
      'There has to be some good thing (masc.).'
  b. *Alguna* buena ha de haber.  
      'There has to be some good thing (fern.).'
  c. *Algo* bueno ha de haber.  
      'There has to be something good.'
(7) a. Jeremías el noble que ninno se clamó, l Otro igual de aquesti ninguno non asmó (Berceo, Loores).
'Jeremiah, the noble man who declared himself to be a child, to THIS ONE, nobody compared.'
b. Eterna ley del mundo aquesta sea (Manuel Quintana, Poesías).
'Let THIS ONE be an eternal law of the world.'
c. La noche estoy llorando y el día, y solo aquesto es mi contento (Fray Luis de León, Salmos).
'I cry night and day, and only THIS is my contentment.'

Or consider the closely related paradigm constituted by the masculine aquése, the feminine aquésa, and the allegedly neuter aqueso. These are 'emphatic' versions of the medial pronouns in (2). Illustrative uses of these pronouns are in turn provided in (8).

(8) a. Yo creo que soy aquese por quien preguntáis (Lope de Rueda, Eufemia).
'I believe I am THAT ONE about whom you inquire.'
b. Ni pienses que á la muerte tengo miedo, | Que aquessa es de los prósperos temida, (Ercilla, La Araucana).
'And do not think that I fear death, since THAT ONE is feared by the prosperous.'
c. El estaba allí para los socorrer cada que menester lo oviessen, e aquesso mismo el príncipe su fijo (Crónica de Don Alvaro de Luna).
'He was there to aid them whenever they needed it, and (for) THAT, too, his son the prince.'

Although used frequently during the Golden Age of Spanish literature (cf. Cuervo 1886: 595), the forms illustrated in (7) and (8) must have lost their emphatic value by then. Consequently, the phonetic increments over the demonstratives in (2) could not be justified, and the erstwhile emphatic forms thus became obsolete. Notice that by 1628 Quevedo could write in his Cuento de Cuentos about the choice of aqueste over éste that "son infinitas las veces que pudiendo escoger, usamos lo peor."

But three genders are taken to be distinguished in further Ibero-Romance paradigms. Consider first Judeo Spanish. According to Cherezli (1899, 205), Subak (1906, 131), and Nehama (1977), this dialect exhibits three relative pronouns corresponding to English which. They are a masculine cual, a feminine cuala, and a neuter cualo. Thus, the masculine cual may serve as the anaphor of masculine nouns like elemento ‘element’ and país ‘country’ while the feminine cuala may take a feminine noun like importancia ‘importance’ as antecedent. The
neuter *cualo*, on the other hand, may refer to what a country with religious tolerance could allow or to the fact that some immense volumes are written in Hebrew.4

Furthermore, interrogative versions of these three pronouns can be found in Asturian vernaculars according to Neira (1956, §46), Alvarez Fernández-Cañedo (1963, 51), Díaz Castañón (1966, 191), Martínez Alvarez (1967, 83), and Conde (1978, 155). The following examples are taken from Neira (1956, 51).

(9) a. Aquí ta l mió sobrín—¿Cuál?
   ‘Here is my nephew’—‘Which one (masc.)?’

b. Aquí ta la mió muyer.—¿Cuál?
   ‘Here is my wife’—‘Which one (fem.)?’

b. Aquí ta algo que nun me gusta.—¿Cuál?
   ‘Here is something I don’t like’—‘Which?’

Finally, a distinction between masculine, feminine and neuter has also been taken to hold in the universal pronoun of Portuguese, where we find a contrast between masculine *todo*, feminine *toda*, and neuter *tudo*:

(10) a. Todo o que vier será bem recebido.
   ‘Anything (masc.) that may come will be well received.’

b. Toda a que vier será bem recebida.
   ‘Any thing (fem.) that may come will be well received.’

c. Tudo o que vier será bem recebido.
   ‘Anything that may come will be well received.’

According to Williams (1962, 98), this distinction can be found by the thirteenth century. And indirect dialectal evidence can be provided that

4 See for instance the following passage, transcribed and compressed from Wagner (1930, 105–113):

en este tiempo el idioma castellano ya había alcanzado un alto grado en su desarrollo y importancia social-política, le *cualo* sin dúbio fue una consecuencia de la victoria de los españoles sobre los árabos [...] los judíos después de su emigración de España, cortaron cualquiera relación espiritual con aquel país, guardaron en alto grado el español gracias a su vida apartada del ambiente no-judio, y dominada en cada reguardo de la profunda conciencia, *cualo* les fue posible en un país de toleración religiosa [...] los inmensos volúmenes de nuestra literatura de ética religiosa y liturgia están escritos en hebreo, *cualo* ejerció sin dúbio su influencia en primer sobre el lenguaje [...] el tercer elemento componiente del len guaje de los sefardes, *cual* echó profundamente sus raíces en nuestra idioma, es el turco [...] los sefardis [...] entran en el ambiente del país en *cual* viven (Kalmi Baruch “La Lingua de los Sefardim”. *El Mundo Sefardi* I: 20–25, 1923).
this distinction was also in force at the time in Spanish. Thus, in a charter dated 1266 in Avilés (Asturies), Alarcos Llorach (1962, 332) cites an instance of maculine *todu* (cf. *de tudo pedido*), an instance of feminine *toda* (cf. *toda talla*), and an instance of neuter *todo* (cf. *esto todo assi como sobredicho yes*).5

3 A Formal Account of the Spanish Forms

3.1 General Remarks.

It has been claimed that Spanish neuters are never nominal in their anaphora; they simply cannot refer back to nouns. Thus, in his summary of the observations made by Bello a century earlier, Fernández Ramírez (1951, §114) stated that ‘the deixis of *el* (*lo*), like that of all neuter pronouns, is not nominal. The pronoun (*el*)*lo* reproduces a predicate, another neuter pronoun, or alludes to a fact or situation which is known or mentioned in discourse by means of a sentence.’6 Along similar lines, Lenz (1925, §193) had pointed out that ‘the neuter demonstratives never reproduce an isolated concept, but rather refer always to a set of things or ideas, as expressed by the verb with its subjects, complements, and predicative attributes. In other words, it shall be said that the pronominal neuters express a set of objects, qualities, or circumstances.’ Such a view has been sanctioned by the Spanish Royal Academy (cf. Real Academia Española 1973, §2.8.1.e), which states that nonneuters ‘reproduce a term of the context, or the name of something given or alluded to in the situation [...] neuters do not effect this particular kind of reference’.

Evidence for these views is not hard to find. The general inability of neuters to serve as anaphors of countable nouns is shown by the referential contrasts in (11a) and by the overt contrasts in (11b). The corresponding points for uncountable nouns can be illustrated with the contrasts in (12).

(11) a. Tuvimos que buscar el anillo_i ya que éste_j / ésta_k / esto_k estaba perdido.

had+we to look+for the ring since that this+one (masc.) / this one (fem.) / this was lost

‘We had to look for the ring since it was lost.’

b. Aquí está el anillo con el / ?la / ?lo cual tuvimos que trabajar.

5 The expression spelled *tudo* in *de tudo pedido* corresponds phonemically to */todu/* and phonetically to [/tudu/], since a stressed *o* is regularly raised here by a final *u* (cf. Alonso 1962).

6 All the quotations of Spanish and Portuguese grammarians found in this chapter are my translations of the originals.
On Conceptional Neuterality

here is the ring with the+one(masc.) / the+one(fem.) / that WH
had+we to work

'Here is the ring with which we had to work.'

(12) a. Existe fierro en el mundo, pero éste / ésta / esto es escaso.
exists iron in the world, but this+one (masc.) / this+one (fem.)
/ this is scarce

'There is iron in the world, but it is scarce.'

b. Tenemos fierro con el / ?la / ?lo cual trabajar.

have+we iron with the+one(masc.) / the+one(fem.) / that WH
work

'We have iron with which to work.'

Now, it will be recalled that nouns have been claimed to denote
homomorphic subsets of the universe of discourse. It might seem,
therefore, that a straightforward formalization of the contrast between
neuters and nonneuters is readily available: neuters draw their
denotations from the heteromorphic portion of the universe of discourse,
while nonneuters select theirs from the homomorphic portion of thereof.

Positive evidence for this view can be gathered from common
observations made by traditional grammarians. Consider for instance the
observation made in Carvalho (1965, 119) about the neuters' ability to
refer to a 'homogeneous although generally heterogeneous set of
objects'. Or consider the view expressed in Bello (1860, §295) about
conjunction, the simplest way of constructing a heteromorphic individual:

Moreover, if we try to reproduce a set of two or more nouns which signify things
(not people), we can do it very well through neuter [pro]nouns, since it is proper
of them to signify, be it unity, be it collective plurality: [...] "Un solo interés,
una sola acción, un solo enredo, un solo desenlace; eso pide, si ha de ser buena,
toda composición teatral" (Moratin). Eso is un solo interés, una sola acción, etc.
[...] Thus, a set of nouns which signify things is, for the reproduction of ideas,
equivalent to a neuter [pro]noun, even though we could also reproduce them by
the [plural pronouns] ellos or ellas in the appropriate gender.

And the preceding comments made by Lenz concerning the ability of
the neuter to express 'a set of objects, qualities, or circumstances' should be taken in the same light.

Yet, things are not that simple, for it is entirely possible to point to
an atomistic element of the universe and say

(13) Esto me gusta.

'I like this.'

As a matter of fact, the neuter pronoun in (13) can denote any
homomorphic element, be it atomic, atomistic, or atomless.
To provide for a unified account of the facts in (11)–(13) we will once again appeal to the Praguean distinction between the general and the specific meaning of an expression. Equipped with this distinction we may now propose that the general meaning (or Gesamtdeutung) of the neuters is such that it allows them to draw their denotations from the entire universe of discourse. Hence the facts in (13) can be accounted for. But nonneuters draw their denotations from the homomorphic portion of universe. The neuters may therefore develop a specific meaning (or Grundbedeutung) which forces them to restrict their denotational domain to heteromorphic portion of the universe. The facts about conjunction and the contrasts in (11)–(12) are thus accounted for as well.

Since a general meaning constitutes the denotation of an expression which is, so to speak, ‘unperturbed by context’, it makes sense to require that grammars provide the general rather than the specific meanings of their expressions. This decision does not represent any real limitation, however, as either meaning can be deduced from the other by reference to the marked (in this case, nonneuter) member of the relevant opposition.

Let us begin, then, to interpret the pronouns in (2). Before doing so, however, we should remark that the feminines in this table will be left uninterpreted. This again is not a real limitation, as all the nonneuters in (2) can receive the same interpretation (except when they denote animates, in which case the masculine denotes males and the feminine denotes females).

3.2 Stressed Personal Pronouns

We begin with the stressed personal pronouns, which we propose to interpret as indicated in (14). As can be readily seen, the two interpretations included therein involve the notion of (contextual) prominence. No definition of this notion will be attempted here, however, as prominence is clearly independent of conceptional neuterality. Suffice it to say at this point that there are two well known ways in which an individual can become most prominent (in a particular context). One is linguistic, and involves being the individual most recently mentioned in a discourse; the other is nonlinguistic, and involves being the individual pointed at by the speaker.

(14) Stressed Personal Pronouns

a. The neuter ello denotes the most prominent individual of the universe (if the context in which this pronoun is used provides such an individual).
b. The nonneuter él denotes the most prominent homomorphic individual of the universe (if the context in which this pronoun is used provides such an individual).

The stressed personal pronouns of Spanish are thus similar in that both select, if anything, the most prominent individual from a component of (1). They are different, however, as to the components from which they draw their selections. The neuter ello draws elements of $E$, while the nonneuter él draws elements of $H$.

To illustrate the interpretations in (14), let us notice that the nonneuter él can denote an atomistic individual (say, the book with missing pages I just bought). It can also denote an atomless individual (say, the wine we just drank). Interestingly, however, our pronoun cannot denote the heteromeric individual which is constituted by an atomistic individual and an atomless individual (say, the mereological sum of the said book and the said wine). This means that the equalities in (15) will hold for some contexts in which él is used while the inequality in (15) will hold in all such contexts.

(15) \[
[[\text{el}]] = [[\text{the book with missing pages I just bought}]] \\
[[\text{el}]] = [[\text{the wine we just drank}]] \\
[[\text{el}]] \neq [[\text{the book with missing pages I just bought and the wine we just drank}]]
\]

The neuter ello, on the other hand, can denote all of these individuals:

(16) \[
[[\text{ello}]] = [[\text{the book with missing pages I just bought}]] \\
[[\text{ello}]] = [[\text{the wine we just drank}]] \\
[[\text{ello}]] = [[\text{the book with missing pages I just bought and the wine we just drank}]]
\]

An example of the referential contrast found in (15) and (16) is provided by the sentences given in (17): the heteromeric individual bracketed in (17) can only be the antecedent of ello; it cannot be referred to by él.

(17) Aquí está [el libro con páginas de menos que acabo de comprar y el vino que acabamos de beber], pero más vale no hablar de él/ello.

'Here is the book with missing pages I just bought and the wine we just drank, but it is better not to talk about it.'

It bears emphasizing that (14) leaves the denotations of ello and él undefined when context fails to provide most prominent individuals of the appropriate kind.
3.3 Unstressed Personal Pronouns

Let us turn next to the unstressed personal pronouns in (2), which we propose to interpret as indicated in (18).

(18) Unstressed Personal Pronouns

a. The neuter \textit{lo} denotes the function which selects the greatest element of any subset of $E$ which has a greatest element.

b. The nonneuter \textit{el} denotes the function which selects the greatest element of any subset of $H$ which has a greatest element.

The unstressed personal pronouns of Spanish are thus similar in that both select greatest elements from subsets of a component of (1). They are different, however, as to the components from whose subsets they draw their selections. The neuter \textit{lo} draws from within subsets of $E$, while the nonneuter \textit{el} draws from within subsets of $H$.

To illustrate the interpretations in (18), let us consider the adjective \textit{rojo} 'red'. By (18a), the interpretation of \textit{lo} \textit{rojo} 'the red' would be as indicated in (19), where we assume that $[[\textit{rojo}]]$ is a set $R$ which contains a greatest element. As usual, $\Gamma$ stands for the function which selects the greatest element from each subset of the universe which has one.

(19) $[[\textit{lo} \textit{rojo}]] = [[[\textit{lo}]][[\textit{rojo}]]] = \Gamma(R)$

The phrase \textit{lo} \textit{rojo} would thus denote the kind constituted by every red element of the universe—and such would seem to be as desired.

But let us assume a universe in which both atomistic individuals (say, cars) and atomless individuals (say, portions of ink) are red. If $[[\textit{rojo}]]$ contains a greatest element, then it will be heteromeric, and $[[\textit{rojo}]]$ would not be a subset of $H$. By (18b), the noun phrase \textit{el} \textit{rojo} would be semantically illformed. This would again seem to be as desired. But let us assume, alternatively, a universe of discourse in which all red individuals are atomistic (consider for instance discourse about a car race). Or consider a universe of discourse in which all red individuals are atomless (consider for instance discourse about dyes). In either case \textit{rojo} would denote a set $R'$ of homomeric individuals. If $R'$ contains a greatest element, then the noun phrase \textit{el} \textit{rojo} would be semantically wellformed and would denote as follows.\footnote{And the same is true if \textit{el} is analyzed as the definite article and \textit{rojo} is taken to be a noun, a 'nominalized adjective', or the modifier of an 'implicit noun'—at least if every noun denotes a set of homomeric individuals and the Spanish article, like its English counterpart, denotes $\Gamma$ (see the preceding sections on the definite article).}
(20) \[ [[\text{el rojo}]] = [[\text{el}}]]([[\text{rojo}}]]) = \Gamma(R) \]

It should again be emphasized that (18) does not define the values of the functions denoted by \textit{lo} and \textit{el} for sets which lack greatest elements.

### 3.4 Demonstrative Pronouns

As can be gathered from (2), Spanish has proximal, medial, and distal demonstratives. The interpretations we wish to assign to them are given in (21)-(23), where we will say that an individual is proximal if it is close to the speaker, distal if it is distant from him, and medial if it lies between the proximal and the distal individuals. Naturally, what counts as close or distant from the speaker is determined by the speaker himself on a case by case basis. Hence, each of the following interpretations must be relativized to a given speech event. Individuals which are either proximal, medial, or distal will be said to be \textit{situated} (relative to the speaker of a given speech event).

(21) **Proximal Demonstratives**

a. The neuter \textit{esto} denotes a function which selects the most prominent proximal individual of any subset of \(E\) which has a most prominent proximal element.

b. The nonneuter \textit{este} denotes a function which selects the most prominent proximal individual of any subset of \(H\) which has a most prominent proximal element.

(22) **Medial Demonstratives**

a. The neuter \textit{eso} denotes a function which selects the most prominent medial individual of any subset of \(E\) which has a most prominent medial element.

b. The nonneuter \textit{ése} denotes a function which selects the most prominent medial individual of any subset of \(H\) which has a most prominent medial element.

(23) **Distal Demonstratives**

a. The neuter \textit{aquello} denotes a function which selects the most prominent distal individual of any subset of \(E\) which has a most prominent distal element.

b. The nonneuter \textit{aquél} denotes a function which selects the most prominent distal individual of any subset of \(H\) which has a most prominent distal element.

The demonstrative pronouns of Spanish are thus similar in that they select most prominent situated individuals from subsets of components of (1). They are different, however, as to the components from whose
subsets they draw their selections. The neuters draw from within subsets of $E$, while the nonneuters draw from within subsets of $H$.

To illustrate the interpretations in (21)–(23) we will combine the medial demonstrative *eso* with the restrictive relative clause *que es rojo* 'which is red' to yield a noun phrase. Given (22a), these combinations will denote as indicated in (24), where $R$ stands for $[[que es rojo]]$, $M$ represents the set of medial individuals, and $\Pi$ symbolizes the function which selects the most prominent individual (if any) in a set.

$$(24) \quad [[eso \ que \ es \ rojo]] = [[eso]]([[que \ es \ rojo]]) = \Pi(R \cap M)$$

The denotation of *eso que es rojo* will thus be defined in case the set of red medial individuals contains a most prominent element, in which case *eso que es rojo* will correctly pick it out.

But suppose again that all red individuals are homomeric and have a most prominent element. By (22b), this most prominent element will be denoted by *ése que es rojo*. The interpretation of this noun phrase would then proceed as indicated in (25), where $R'$ is a set of homomeric individuals.

$$(25) \quad [[ése \ que \ es \ rojo]] = [[ése]]([[que \ es \ rojo]]) = \Pi(R' \cap M)$$

Notice that even though the individual denoted by *ése que es rojo* will have to be homomeric, it may be either atomistic or atomless.

Notice now that stressed personal pronouns are independent in the sense that they may denote without having to combine with restrictive modifiers; they occur in what traditional grammars call 'absolute constructions'. Unstressed personal pronouns, on the other hand, are dependent in that they have to combine with a restrictive modifier in order to denote. Demonstrative pronouns are like personal pronouns in that they too may be used either dependently or independently. The phrases interpreted in (24) and (25) illustrate the dependent uses of demonstratives; the sentences in (5), which may be uttered while pointing at something, illustrate the independent uses of demonstratives.

True, the personal paradigm distinguishes between dependent and independent forms. The demonstrative paradigm, on the other hand, distinguishes only between dependent and independent uses of the same forms. Such uses, however, are no less real, and must also be accounted for. To do so we will distinguish between the denotation of our demonstratives as pronouns and their denotations as noun phrases. This distinction is entirely analogous to the one drawn in Chapter 5 between

8 Dependent pronouns need not be identified with determiners, which have to combine with nouns (or nominals) in order to denote.
the denotation of an uncountable as a noun and its denotation as a noun phrase.

To be more specific, let us say that the 'lexical' denotation of our pronouns is the one detailed in (21)–(23), whereas the 'maximal' denotation of our pronouns can be derived from the former by a simple procedure illustrated in (26) for the proximal pronouns. The terms 'lexical' and 'maximal' are of course syntactic in origin, and allude to the 'projection' (or 'bar') level of nouns and noun phrases, respectively.9

\[(26)\]
\[
\left[esto_{\text{MAX}}\right] = \left[esto_{\text{LEX}}\right](E)
\]
\[
\left[\acute{e}ste_{\text{MAX}}\right] = \left[\acute{e}ste_{\text{LEX}}\right](H)
\]

More generally, the maximal denotation of a demonstrative is the result of applying its lexical denotation to \(E\) (if neuter) or to \(H\) (if nonneuter). As a result of this, demonstratives used independently will denote the most prominent situated individuals of the universe (if neuter) or of the homomeric portion thereof (if nonneuter). It follows that the sentence in (5c), uttered when pointing at some medially situated individual, will succeed in predicating heaviness of it. As to (5a), it will so succeed when uttered under the same circumstances—provided that the individual pointed at is homomeric.

It should be pointed out that the proposed account of the independent uses of demonstratives allows us to explain their independence. For notice that there is a clear sense in which the set \(E\) is 'given with the neuter demonstratives' while the set \(H\) is 'given with the nonneuter demonstratives'. As a consequence of this, these sets need not be mentioned, and demonstratives which select from these sets may indeed occur in the absolute construction.

3.5 Existential Pronouns

So far, the difference between the neuter and the nonneuter has pertained to the difference between a universe and its homomeric portion. The difference between the neuter and the nonneuter within the existential pronouns is, however, starker, as it pertains to the difference between a universe and its atomic portion. To illustrate this point let us consider the ability of the personal pronoun \(\acute{e}l\) to refer either to an atomic individual like a ring (27a), or to an atomless individual like a (contextually prominent) portion of gold (27b).

\[(27)\]
\[a. \text{Aquí tiene que estar el anillo}_i; \text{hasta ahora no hemos podido dar con } \acute{e}l_i.\]

9 See Kornai & Pullum (1990) for a recent discussion of X-bar theory.
here has to be the ring I until now not have+we been+able give with it
'The ring has to be here; until now we have not been able to hit upon it'.
b. Aquí tiene que estar el oro; hasta ahora no hemos podido dar con él.
here has to be the gold I until now not have+we been+able give with it
'The gold must be here; until now we have not been able to hit upon it.'

The referential powers of él thus contrast with those of alguno, its existential counterpart in (2), as the latter may refer back to some rings (28a), but not to some gold (28b).

(28) a. Aquí deben estar los anillos; al menos alguno.
here must be the rings I at+the least some+one
'The rings must be here; at least one.'
b. Aquí debe estar el oro; al menos alguno.
here must be the gold I at+the least some+one

If reference to some gold is intended, then algo, the neuter existential pronoun is called for. Consider for instance (28c).

c. Aquí debe estar el oro; al menos algo.
here must be the gold I at+the least some
'The gold must be here; at least some (of it).'</n
We should therefore want to interpret the existential pronouns in (2) as indicated in (29).

(29) Existential Pronouns
a. The neuter algo denotes a function which assigns, to each \( P \subseteq E \), the family \( \{ X \subseteq E : X \cap P \neq \emptyset \} \).
b. The nonneuter alguno denotes a function which assigns, to each \( P \subseteq A \), the family \( \{ X \subseteq E : X \cap P \neq \emptyset \} \).

The existential pronouns of Spanish are thus similar in that they identify the conjoints of subsets of a component of (1).\(^{10}\) They are different, however, as to the components whose subsets they identify the conjoints of. The neuter algo identifies the conjoints of subsets of \( E \), while the nonneuter alguno identifies the conjoints of subsets of \( A \).

To illustrate the interpretations in (29) we may combine the neuter algo with the prepositional phrase de color rojo 'of red color' to yield a

\(^{10}\) The notion of conjoint has been defined in Section 13 of Chapter 3.
noun phrase. Given (29a), this combination will denote as indicated in (30), where $R$ stands for $\llbracket \text{de color rojo} \rrbracket$.

(30) \[ \llbracket \text{algo de color rojo} \rrbracket = \llbracket \text{algo} \rrbracket (\llbracket \text{de color rojo} \rrbracket) = \{ X \subseteq E : X \cap R \neq \emptyset \} \]

The denotation of \text{algo de color rojo} will thus be, appropriately, the family of subsets of the universe which overlap with $R$.

But suppose once again that all the red individuals of the universe are atomic (say they are cars). By (29b), \text{alguno de color rojo} would now be the family of subsets which overlap with $R'$, the set of red individuals, all of which are atomic.

(31) \[ \llbracket \text{alguno de color rojo} \rrbracket = \llbracket \text{alguno} \rrbracket (\llbracket \text{de color rojo} \rrbracket) = \{ X \subseteq E : X \cap R' \neq \emptyset \} \]

But notice now that existential pronouns, like their personal and demonstrative counterparts, may be used independently as well as dependently. Instances of the latter have been already interpreted in (30)--(31). Examples of the former are rather easy to find:

(32) a. Algo nos habla de Dios.
   something to+us speaks of God
   'Something speaks to us about God.'

   b. Alguno nos habla de Dios.
   some+thing to+us speaks of God
   'Some thing speaks to us about God.'

To account for the independent uses of existential pronouns we will resort to the same simple procedure invoked in our account of the independent uses of demonstratives. We will therefore regard (29) as the 'lexical' interpretations of the existential pronouns in (2), and derive their 'maximal' interpretations as follows:

(33) a. \[ \llbracket \text{algo}_{\text{MAX}} \rrbracket = \llbracket \text{algo}_{\text{LEX}} \rrbracket (E) = \{ X \subseteq E : X \cap E \neq \emptyset \} = \{ X \subseteq E : X \neq \emptyset \} \]

   b. \[ \llbracket \text{alguno}_{\text{MAX}} \rrbracket = \llbracket \text{alguno}_{\text{LEX}} \rrbracket (A) = \{ X \subseteq E : X \cap A \neq \emptyset \} \]

When taken 'maximally', the existential pronouns are therefore similar in that they denote the conjoints of a component of (1). They are different, however, as to the components they can identify the conjoints of. Thus, \text{algo} identifies the conjoints of the universe (which is, in fact, the family of nonempty subsets of the universe) while \text{alguno} identifies the conjoints of the atomic portion of the universe.

To illustrate the interpretations in (33) we will once again assume that a sentence is true just in case the set denoted by its predicate is an
element of the family of sets denoted by its subject. Equipped with (33a) and this assumption, the sentence in (32a) denotes as follows.

\[(34) \quad [[\text{algo nos habla de Dios}]] = [[\text{nos habla de Dios}}] \in [[\text{algo}}] =
\]
\[[[\text{nos habla de Dios}]] \in \{X \subseteq E : X \neq \emptyset\}\]

It follows that (32a) is tantamount to the assertion that the set of individuals that speak to us about God is not empty (or that there is an individual that speaks to us about God). Judging from our intuitions about (32a), this should be as desired.

But consider next the sentence in (32b). Equipped with (33b) and the usual sentential semantics, this sentence would be interpreted as indicated in (35).

\[(35) \quad [[\text{alguno nos habla de Dios}]] = [[\text{nos habla de Dios}}] \in [[\text{alguno}}] =
\]
\[[[\text{nos habla de Dios}]] \in \{X \subseteq E : X \cap A \neq \emptyset\}\]

The sentence in (32b) is thus tantamount to the assertion that the set of atomic individuals that speak to us about God is not empty (or that there is an atomic individual that speaks to us about God).

It should be clear that if we focus on the independent uses of the existential pronouns, the family of sets denoted by \text{algo} is invariably more inclusive than the one denoted by \text{alguno}:

\[(36) \quad [[\text{alguno}}] \subseteq [[\text{algo}}]\]

As a consequence of this, every statement about \text{alguno} will entail the corresponding statement about \text{algo} (though not conversely):

\[(37) \quad [[\text{alguno}}](P) \rightarrow [[\text{algo}}](P)\]

Thus, the sentence in (32b) entails the sentence in (32a), though not conversely.

3.6 Nonexistential Pronouns

Let us turn finally to the nonexistential pronouns in (2). We begin by pointing out that the difference between the neuter and the nonneuter within this paradigm pertains to the difference between the universe and the atomic portion thereof. In this the nonexistential pronouns are like the existential pronouns and unlike the personal pronouns. To illustrate these points let us recall that the personal pronoun \text{él} was able to refer either to an atomic individual like a ring (27a), or to an atomless individual like a portion of gold (27b). The nonexistential \text{ninguno}, on the other hand, may only refer to the rings; it may not refer to the gold:

\[(38) \quad \text{a. Aquí deben estar los anillos; hasta ahora no hemos podido dar con ninguno.}\]
here must be the rings I until now not have+we been+able give with none
'The rings must be here; until now we have not been able to hit upon one.'

b. ?Aquí debe estar el oro; hasta ahora no hemos podido dar con ninguno.
here must be the gold I until now not have+we been+able give with none
'The gold must be here; until now we have not been able to hit upon any (of it).'

When reference to some gold is intended, then nada, the neuter nonexistential is called for. See for instance (38c).

c. Aquí debe estar el oro; hasta ahora no hemos podido dar con nada.
here must be the gold I until now not have+we been+able give with none
'The gold must be here; until now we have not been able to hit upon any (of it).'

We should therefore want to interpret the nonexistential pronouns in (2) as indicated in (39).

(39) Nonexistential Pronouns

a. The neuter nada denotes a function which assigns, to each $P \subseteq E$, the family \( \{X \subseteq E : X \cap P = \emptyset\} \).

b. The nonneuter ninguno denotes a function which assigns, to each $P \subseteq A$, the family \( \{X \subseteq E : X \cap P = \emptyset\} \).

The nonexistential pronouns of Spanish are thus similar in that they identify the disjoints of subsets of a component of (1). They are different, however, as to the components whose subsets they identify the disjoints of. The neuter nada identifies the disjoints of subsets of $E$, while the nonneuter ninguno identifies the disjoints of subsets of $A$.

To illustrate the interpretations in (39) we may combine the neuter nada with the prepositional phrase de color rojo 'of red color' to yield a noun phrase. Given (39a), this combination will denote as indicated in (40), where $R$ stands for \([ \text{de color rojo} ]\).

(40) \([\text{nada de color rojo}] = [\text{nada}]( [\text{de color rojo}]) = \{X \subseteq E : X \cap R = \emptyset\}\)

The denotation of nada de color rojo will thus be, appropriately, the family of subsets of the universe which are disjoint with $R$.

But suppose once again that all the red individuals of the universe are atomic (say they are cars). By (39b), ninguno de color rojo would

---

11 The notion of disjoint has been defined in Section 14 of Chapter 3.
now be the family of subsets which are disjoint with \( R' \), the set of red individuals, all of which are atomic.

\[
\text{(41) } [\text{nobody of color red}] = [\text{nobody}](\text{of color red}) = \{X \subseteq E: X \cap R' = \emptyset\}
\]

But notice now that nonexistent pronouns, like their personal, demonstrative, and existential counterparts, may be used independently as well as dependently. Instances of the latter have been already interpreted in (40)–(41). Examples of the former can be provided by the sentences in (42), a simple variation of a previous example.

(42) a. Nada nos habla de Dios.
    nothing to+us speaks of God
    ‘Nothing speaks to us about God.’

b. Ninguno nos habla de Dios.
    no+thing to+us speaks of God
    ‘No thing speaks to us about God.’

To account for the independent uses of nonexistent pronouns we will resort to the usual procedure: we will regard (39) as the ‘lexical’ interpretations of the nonexistent pronouns in (2), and derive their ‘maximal’ interpretations as follows:

(43) a. \( [\text{nada}_{\text{MAX}}] = [\text{nada}_{\text{LEX}}](E) = \{X \subseteq E: X \cap E = \emptyset\} = \{\emptyset\} \)

b. \( [\text{ninguno}_{\text{MAX}}] = [\text{ninguno}_{\text{LEX}}](A) = \{X \subseteq E: X \cap A = \emptyset\} \)

When taken ‘maximally’, the nonexistent pronouns are therefore similar in that they can identify the disjoints of a component of (1). They are different, however, as to the components they can identify the disjoints of. Thus, \text{nada} identifies the disjoints of the universe (there is only one such disjoint, namely, the empty set) while \text{alguno} identifies the disjoints of the atomic portion of the universe.

To illustrate the interpretations in (43) we may return first to the sentence in (42a). Given (43a) and the usual sentential semantics, our sentence denotes as follows.

(44) \( [\text{nada nos habla de Dios}] = [\text{nos habla de Dios}] \in [\text{nada}] = [\text{nos habla de Dios}] \in \{\emptyset\} \)

Since \( \{\emptyset\} \) is a singleton, it follows that (42a) states that \( [\text{nos habla de Dios}] = \emptyset \). But this means that (32) asserts that the set of individuals that speak to us about God is empty (or that there is no individual that speaks to us about God). This again is in accordance with intuition.

But consider next the sentence in (42b). Equipped with (43b) and the usual sentential semantics, this sentence would be interpreted as indicated in (45).
(45) \[
\text{[[ninguno nos habla de Dios]]} = \text{[[nos habla de Dios]]} \in \\
\text{[[ninguno]]} = \text{[[nos habla de Dios]]} \in \{X \subseteq E : X \cap A = \emptyset\}
\]
The sentence in (42b) is thus tantamount to the assertion that the set of atomic individuals that speak to us about God is empty (or that there is no atomic individual that speaks to us about God).

It should be clear that if we focus on the independent uses of the nonexistential pronouns, the family of sets denoted by \text{ninguno} is invariably more inclusive than the one denoted by \text{nada}:

(46) \[
\text{[[nada]]} \subseteq \text{[[ninguno]]}
\]
As a consequence of this, every statement about \text{nada} will entail the corresponding statement about \text{ninguno} (though not conversely):

(47) \[
\text{[[nada]]}(P) \rightarrow \text{[[ninguno]]}(P)
\]
Thus, the sentence in (42a) entails the sentence in (42b), though not conversely.

4 Conceptional Neuterality: A Synthesis

To summarize, then, we have provided the conceptional neutrals of Spanish with formal interpretations as indicated in (48). The denotation of \text{ello} (in a given context) is the value of \text{\Pi}, the prominence function, on the universe \text{E} (and the context). The denotation of \text{lo} is the restriction of \text{\Gamma}, the greatest function, to \text{P(E)}, the family of subsets of \text{E}. The denotation of \text{esto} (in a given context) is the restriction of the prominence function to the family of proximal conjoints of \text{E} (and the context). \text{Eso} and \text{aquello} differ from \text{esto} in that they involve the medial and distal conjoints of \text{E} respectively. Finally, the denotation of \text{algo} is the restriction of \text{\Delta}, the disjoint function, to \text{P(E)}; the denotation of \text{nada} is the restriction of \text{\Delta} the disjoint function, to \text{P(E)}.

\begin{align*}
(48) \quad & a. \text{[[ello]]} = \Pi(E) \\
& b. \text{[[lo]]} = \Gamma \setminus P(E) \\
& c. \text{[[esto]]} = \Pi \setminus P(Q \cap E) \\
& d. \text{[[eso]]} = \Pi \setminus P(N \cap E) \\
& e. \text{[[aquello]]} = \Pi \setminus P(D \cap E) \\
& f. \text{[[algo]]} = K \setminus P(E) \\
& g. \text{[[nada]]} = \Delta \setminus P(E)
\end{align*}

But a list of interpretations is not the identification of a distinctive semantic trait. To provide a linguistic category with a truly general

\[\text{12 As might be expected, the conjoint (or disjoint) function is the function which assigns, to each subset of the universe, the set of conjoints (respectively disjoints) of the subset in the universe. See Sections 13 and 14 of Chapter 3.}\]
semantic characterization we need to identify the semantic property which brings this category together and sets it apart from the rest.

To provide a truly general semantic characterization of the Spanish neuter we should consider (49), the formal interpretations we have assigned to the nonneuters in (2).

(49) a. \[
\text{[[} \text{él}] = \Pi (H)\]
b. \[
\text{[[el]] = } \Gamma | \mathcal{P}(H)\]
c. \[
\text{[[éste]] = } \Pi | \mathcal{P}(Q \cap H)\]
d. \[
\text{[[ése]] = } \Pi | \mathcal{P}(N \cap H)\]
e. \[
\text{[[aquél]] = } \Pi | \mathcal{P}(D \cap H)\]
f. \[
\text{[[alguno]] = } K | \mathcal{P}(A)\]
g. \[
\text{[[ninguno]] = } \Delta | \mathcal{P}(A)\]

As can be readily seen, the nonneuters involve the homomeric (or the atomic) portion of the universe where the neuters involve the entire universe.

To make this more precise, let us say that linguistic theory provides us with four 'mereological features', namely, INDIVIDUAL, HOMOMERIC, ATOMISTIC, and ATOMIC. Let us moreover assume that these mereological features are binary, and that their specifications denote the components of (1) in the obvious way. Thus, we assume that

(49) \[
\text{[[+INDIVIDUAL]] = } E
\]
\[
\text{[[+ATOMISTIC]] = } M
\]
\[
\text{[[+HOMOMERIC]] = } H
\]
\[
\text{[[+ATOMIC]] = } A
\]

(as to the negative specifications for these features, they may be taken to denote the difference between the universe and the denotation of the positive specification). Now, if the lexical entry of a pronoun must be specified for one (and only one) mereological feature, we may then say that the Spanish neuter is the class of [+INDIVIDUAL] pronouns. It is the class of pronouns whose mereological component is the entire universe of discourse. As to the nonneuter, it is subject to paradigmatic variation. The definite is [+HOMOMERIC] whereas the indefinite is [+ATOMIC].

13 To illustrate how the [+INDIVIDUAL] specification of a neuter pronoun may combine with the other semantic components of the pronoun, let us say that linguistic theory in addition provides two semantic features, namely DEMONSTRATIVE and MEDIAL. Let us moreover assume

\[
\text{[[+DEMONSTRATIVE]] = } \lambda X[\lambda Y[\Pi | \mathcal{P}(X \cap Y)]]
\]
\[
\text{[[+MEDIAL]] = } N
\]
We opened this chapter by presenting the reader with the conceptional neuter, a perplexing category, glimpses of which have been provided by Jespersen, Brøndal, Priestly, and Klajn. Based on our study of Spanish, we have proposed that the conceptional neuter is simply [+PRONOMINAL, +INDIVIDUAL] the class of pronouns whose mereological component is the entire universe of discourse. The argumentation leading to this conclusion assumed crucially the typology of kinds diagramed in (1). This typology was in turn based on the central thesis of this study—namely that any model which is fit for doing natural language semantics must provide a set of individuals and a relation of instantiation which jointly constitute a mereology.

The interpretation of eso can now be rendered not only compositional, but furthermore type-driven:

\[
\llbracket \text{eso} \rrbracket = \llbracket +\text{DEMONSTRATIVE}, +\text{MEDIAL}, +\text{INDIVIDUAL} \rrbracket \\
= \lambda x[\lambda y[\Pi | P(x \cap y)](N)(E)] \\
= \lambda y[\Pi | P(N \cap y)](E) = \Pi | P(N \cap E)
\]
Appendix A

The Boolean Theory of Individuals

It was claimed in Link (1983) that the set of individuals provided by an admissible model must have the structure of a boolean algebra which is both complete and atomic, some of whose atoms constitute a complete join semilattice which is a homomorphic image (as a semilattice) of the positive portion of the entire boolean algebra. The purpose of this Appendix is to spell out these compactly formulated proposals.

To say that the set (call it $E$) of individuals provided by an admissible model must have the structure of a boolean algebra is to say that there are at least two elements in $E$ (call them $i$ and $o$), that there are two binary operations on $E$ (call them $\wedge$ and $\vee$), and that there is one singularly operation on $E$ (call it $'$) which jointly satisfy the eight postulates in (1), wherein $x, y, z$ are arbitrary members of $E$.

\begin{align*}
(1) \quad \text{COMMUTATIVITY:} & \quad x \wedge y = y \wedge x \\
& \quad x \vee y = y \vee x \\
\text{DISTRIBUTIVITY:} & \quad x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) \\
& \quad x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z) \\
\text{IDENTITY:} & \quad x \wedge i = x \\
& \quad x \vee o = o \\
\text{INVERSES:} & \quad x \vee x' = i \\
& \quad x \wedge x' = o
\end{align*}

To say that $E$ has the structure of a complete boolean algebra is to say that $E$ has the structure of a boolean algebra which moreover satisfies the postulates in (2) when taken in conjunction with the binary operations of the algebra.

\begin{align*}
(2) \quad \text{LEAST UPPER BOUNDEDNESS:} & \quad \text{For each } F \subseteq E \text{ there is an element} \\
& \quad \text{of } E, \text{ call it } \Sigma(F), \text{ such that} \\
& \quad (i) \quad \text{For all } x \in F: \quad x \vee \Sigma(F) = \Sigma(F) \\
& \quad (ii) \quad \text{For all } x \in F: y \in E \text{ and } x \vee y = y \text{ jointly imply that } \Sigma(F) \wedge y \\
& \quad \quad = \Sigma(F).
\end{align*}

---

1 A singulary operation on a set $E$ is simply a function from $E$ to $E$. A binary operation on a set $E$ is a function from the set $E \times E$ to the set $E$. 

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GREATEST LOWER BOUNDEDNESS: For each $F \subseteq E$ there is an element of $E$, call it $\Pi(F)$, so that

(i) For all $x \in F$: $x \land \Pi(F) = \Pi(F)$

(ii) For all $x \in F$: $y \in E$ and $x \land y = y$ jointly imply that $\Pi(F) \lor y = \Pi(F)$.

The elements $\Sigma(F)$ are said to be least upper bounds relative to the boolean algebra. Similarly, the elements $\Pi(F)$ are said to be greatest lower bounds relative to the boolean algebra.

Next, to say that $E$ has the structure of a complete and atomic boolean algebra is to say that $E$ has the structure of a complete boolean algebra which moreover satisfies the postulate in (3).

(3) ATOMICITY: For each $x \in E$ other than $0$ there is some atom $a$ such that $x \land a = a$.

Next, to say that $E$ has the structure of a boolean algebra which is both complete and atomic, some of whose atoms constitute a complete join semilattice is to say that $E$ has the structure of a boolean algebra which satisfies postulates (2)–(3) and that there is a nonempty set $D$ of atoms of $E$ which satisfies the postulates in (4) and (5) when taken in conjunction with a binary relation $\leq$ over the elements of $D$. Here again $x, y, z$ are arbitrary members of $D$.

(4) REFLEXIVITY: $x \leq x$.

ANTISYMMETRY: $x \leq y$ and $y \leq x$ jointly imply that $x = y$.

TRANSITIVITY: $x \leq y$ and $y \leq z$ jointly imply that $x \leq z$.

(5) COMPLETENESS: For each nonempty $C \subseteq D$ there is an element of $D$, call it $\Sigma(C)$, such that

(i) For all $x \in C$: $x \leq \Sigma(C)$.

(ii) For all $x \in C$: $y \in D$ and $x \leq y$ jointly imply that $\Sigma(C) \leq y$.

The elements $\Sigma(C)$ are said to be least upper bounds relative to the join semilattice.

Finally, then, to say that $E$ has the structure of a boolean algebra which is both complete and atomic, some of whose atoms constitute a complete join semilattice which is a homomorphic image (as a semilattice) of the positive portion of the entire boolean algebra is to say that there exists a function $h$ from $E - \{0\}$ to $D$ such that

(6) For all $x \in D$: $h(x) = x$.

(7) For any subset $C$ of $E - \{0\}$: $h(\Sigma(C)) = \Sigma(h[C])$.

---

2 An $a \in E$ is an atom if and only if for all $x \in E$: $a \land x = a$ implies either $x = a$ or $x = 0$. 
Notice that 'Σ' is used ambiguously in (7). Σ(C) is a least upper bound relative to the entire boolean algebra. Σ(h[C]) is a least upper bound relative to the join semilattice. Notice also that h[C] = \{ h(x): x \in C \}. Notice finally, that when the set D is so constrained, it constitutes a mereology under ≤.

As Link would have it, then, a universe of discourse is to be partitioned as diagramed in (8), where E is the set of individuals of the universe of discourse. As indicated above, E has the structure of a complete and atomic boolean algebra, A is the set of atoms of this algebra, and D is a set of atoms which is also a complete join semilattice in its own right.

(8)

```
    E
   / \  \
A   E - A
  /      /
D  A - D
```
Appendix B

On the Construction of Bells

How many ways are there to split a set into disjoint subsets? If the set has one element, then there will be just one. If the set has two elements, then there will be two. If the set has three, then there will be five. As the number of elements of the set increases, a sequence \(<1, 2, 5, 15, 52, \ldots>\) is being generated. The members of this series are called the Bell numbers, after Eric Temple Bell, the first mathematician to study them in depth.

Formulas for constructing the \(n\)th Bell number have been found. The Dobinski formula for the \(n\)th Bell number, for instance, is as follows.\(^1\)

\[
B(n) = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}
\]

It will be noticed that this formula involves the addition of infinitely many terms. One formula which doesn’t is the following. It provides the \(n+1\)th Bell number.\(^2\)

\[
B(n+1) = \sum_{k=0}^{n} C(n, k) B(k)
\]

But this formula is far from providing us with computational ease (try to construct the sixth Bell from it!). Fortunately, there is a very simple algorithmic procedure for constructing the \(n\)th Bell number. It involves constructing the following ‘triangle of numbers’.

1
1 2
2 3 5
5 7 10 15
15 20 27 37 52
\ldots \ldots \ldots \ldots \ldots \ldots

---

\(^1\) Here \(e\) is the transcendental Euler number (\(e = 2.72\)).
\(^2\) \(C(n, k)\) is a binomial number, so that \(C(n, k) = n!/(k!(n-k)!). By convention, B(0) = 1.\)
To construct this triangle, we proceed line by line, from top to bottom. We begin by letting '1' constitute the top line. Now, we follow two simple rules. The first is that the first element of a new line is the last element of the preceding one (so 1 is the first element of the second line). The second is that each one of the remaining elements of a new line is the sum of the preceding one plus the element directly above it (so the next element of the second line is 2, since it is the sum of 1 plus 1). A line is completed whenever you reach a number without a top neighbor (so 2 is the last element of the second line). The $n$th Bell number is the $n$th number down the hypothenuse of the triangle.$^3$

Bell numbers play an important role in number theory, where they count the number of factorizations of a square free number (a number is square free if it is not divisible by any square other than 1). For indeed, the number of factorizations of a square free number with $n$ prime factors is $B(n)$. Consider for instance 30. It is square free, since its prime divisors are 2, 3, 5. It therefore has $B(3) = 5$ factorizations. They are $2 \times 3 \times 5, 2 \times 15, 3 \times 10, 5 \times 6, 30.$\(^4\)

But it will be recalled that the sequence $<2, 3, 5>$ of primes generates a mereology under the relation of divisibility. More specifically, it will generate the mereology with the following Hasse diagram.

![Hasse diagram](image)

Notice that this mereology has five nonempty submereologies. They are the ones generated by $\{2, 3, 5\}$, $\{2, 15\}$, $\{3, 10\}$, $\{5, 6\}$, and $\{30\}$. It should not escape the reader that there is one submereology for each factorization of 30, and that these submereologies are all atomistic. In fact, each set of factors is the set of atoms of some submereology and, conversely, each one of these sets of atoms is a set of factors.

---

$^3$ At least if $n \geq 1$. If $n \geq 0$, then it is the $n$th number down the vertical cathetus.

$^4$ Much of the preceding has been taken from Gardner (1978). For more on the Bell numbers, see the references under sequence 585 in Sloane (1973, 71).
But we have seen in Chapter 2 that every sequence $<2, 3, 5, \ldots, n>$ of primes will generate a mereology. What we said about $<2, 3, 5>$ extends to all of them. This means that any mereology generated by a sequence $<2, 3, 5, \ldots, n>$ of primes will have $B(n)$ (nonempty) submereologies. These submereologies will be atomistic, and $A$ will be a set of factors of $n$ if and only if $A$ is a set of atoms of a submereology.

We have also seen in Chapter 2 that the sequences $<2, 3, 5, \ldots, n>$ of primes are illustrative of the entire family of finite mereologies (every finite mereology is isomorphic to the mereology generated by some sequence of primes). Hence what we have said about the mereologies generated by a sequence $<2, 3, 5, \ldots, n>$ of primes extends to all finite mereologies.

Now, we have claimed in Chapter 4 that every countable nominal stem denotes an atomistic mereology. So if restrict our attention to finite models, every countable stem denotation will be isomorphic to the mereology generated by some sequence $<2, 3, 5, \ldots, n>$ of primes, and will therefore have exactly $B(n)$ nonempty submereologies. But we have also claimed in Chapter 4 that if $N$ is a stem which denotes some mereology $M$, then each submereology of $M$ is a possible sense of $N$. It follows then, as claimed in Chapter 4, that the Bells will be able to count the number of mereologically distinct (nonvacuous) senses of a countable stem in a finite universe.

Let us consider now the family of nonempty subsets of some set $A$. We have seen that any subset of $A$ which has $k$ elements will have $B(k)$ partitions. And if $A$ has $n$ elements, then there will be $C(n, k)$ subsets of $A$ which in fact have $k$ elements. Hence the total number of partitions contained in our family of sets will be

$$\sum_{k=0}^{n} C(n, k) B(k)$$

But by the second formula in this appendix, this number will be $B(n+1)$. Now, we have seen in Chapter 2 that the family of nonempty subsets of a set with $n$ elements constitutes a mereology. In fact, the mereology will be atomistic, and will have $n$ atoms. $B(n+1)$ is therefore the number of (nonempty) pairwise disjoint subsets of this atomistic mereology. Let us say now that this atomistic mereology, call it $E$, is our universe of discourse. In light of the constraints in Chapter 4, there will be as many (nonempty) pairwise disjoint subsets of $E$ as there are potential nonvacuous countable stem denotations in $E$. It follows that $B(n+1)$ is, as claimed in Chapter 4, the number of nonvacuous countable stems which a model with an atomistic universe of $n$ atoms may interpret.
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